



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

Wentworth · Answers to Algebra

1899

EducT

128

99.916



3 2044 097 012 702

Educ T 128.99.916



HARVARD
COLLEGE
LIBRARY

THE GIFT OF

*Miss Ellen Lang Wentworth
of Exeter, New Hampshire*

° ANSWERS

TO THE

NEW SCHOOL ALGEBRA

BY

G. A. WENTWORTH

BOSTON, U.S.A.

GINN & COMPANY, PUBLISHERS

1899

✓
Edw T. 128.99.916

HARVARD COLLEGE LIBRARY
GIFT OF
MISS ELLEN L. WENTWORTH

MAY 8 1939

ANSWERS.

Exercise 1. Page 9.

1. 75.	7. 900.	13. 54.	19. 360.	25. 10.	31. 10.
2. 6.	8. 320.	14. 432.	20. 336.	26. 12.	32. 0.
3. 84.	9. 432.	15. 0.	21. 1.	27. 10.	33. 6.
4. 40.	10. 72.	16. 720.	22. 7.	28. 6.	34. 24.
5. 324.	11. 27.	17. 375.	23. 5.	29. 10.	35. 30.
6. 36.	12. 216.	18. 0.	24. 0.	30. 6.	

Exercise 2. Page 10.

1. 1.	6. 5.	11. 19.	16. 13.	21. 8.	26. 8.
2. 5.	7. 14.	12. 2.	17. 12.	22. 0.	27. 5.
3. 5.	8. 86.	13. 12.	18. 0.	23. 8.	28. 74.
4. 7.	9. 18.	14. 25.	19. 1.	24. 4.	29. $6\frac{1}{2}$.
5. 3.	10. 2.	15. 1.	20. 5.	25. 8.	30. 17.

Exercise 3. Page 13.

1. 14.	3. 13.	5. 7.	7. 2.	9. 0.	11. 4.	13. 21.	15. 3.
2. 10.	4. 11.	6. 6.	8. 3.	10. 3.	12. 10.	14. 13.	16. 7.

Exercise 4. Page 14.

1. 69.	4. 55.	7. 17.	10. 35.	13. 3.	16. 5.
2. 76.	5. 49.	8. 8.	11. 14.	14. 11.	17. 0.
3. 34.	6. 15.	9. 1.	12. 24.	15. 13.	18. 2.

Exercise 5. Page 14.

1. 3.	3. 1.	5. 17.	7. 3.	9. $2\frac{1}{2}$.	11. 15.
2. $2\frac{1}{2}$.	4. 1.	6. 5.	8. 15.	10. 3.	12. 3.

Exercise 6. Page 19.

1. 4.	7. 4.	13. 4.	18. 5.	23. 41.	28. 5.
2. 7.	8. 4.	14. 12.	19. 1.	24. 7.	29. 6.
3. 2.	9. 11.	15. 4.	20. 2.	25. 6.	30. 5.
4. 9.	10. 9.	16. 4.	21. 18.	26. 5.	31. 16.
5. 11.	11. 2.	17. 3.	22. 2.	27. 6.	32. 6.
6. 3.	12. 5.				

Exercise 7. Page 20.

1. $a - b$; $a + b$; ab ; $\frac{a}{b}$; a^2 ; $\sqrt{a}$; $\sqrt[3]{a}$; a^2b^4 .
2. $4x$; ax .
3. $\frac{12}{3}$; $\frac{x}{3}$.
4. $\frac{20}{x}$; $\frac{y}{x}$.
5. $\frac{20}{8}$; $\frac{a}{b}$.
6. $4 \times 3 + 2$; $dq + r$.
7. $\frac{22-2}{4}$; $\frac{p-r}{d}$.
8. $\frac{22-2}{4}$; $\frac{p-r}{q}$.
9. $25 - 10$.
10. $30 - x$.
11. $x - c$.
12. $40 - 25$.
13. $x - 5$.
14. $s - a$.
15. $13 + 7$.
16. $x + a$.
17. $x - c$.
18. $a - 4$; $a + 4$.
19. $x - b$; $x + a$.
20. $5x - 3x$.
21. $x - a$.
22. $x - a$.
23. $2x + 3 - (x + 1)$.
24. $3x - 4(18 - x)$.
25. $x - 50 - (80 - x)$.
26. $2x - 24 - (80 - 5x)$.
27. $5x + 24 - (60 - x)$.
28. $50a + 25b$.
29. $100a + 10b + c$.
30. $100a - 50b - 10c$.
31. $100x + 10y + z - 50a - 25b$.
32. $(236 - a - c)$ miles.
33. ma .
34. $\frac{a^2}{9}$.
35. $\frac{ab}{9}$.
36. $\frac{ab - c^2}{9}$.

Exercise 8. Page 26.

1. 90.
2. Father, 49 years; son, 7 years.
3. 138; 23.
4. Left standing, 10 feet; broken off, 90 feet.
5. 35; 28.
6. 36; 23.
7. 21; 15.
8. 18.
9. 10.
10. 32; 8.
11. 27; 21.
12. 35; 7.
13. 35; 36; 37.
14. 12; 13; 14; 15; 16.
15. 48.
16. 16.
17. 7.
18. 35.
19. 5.
20. 12.
21. 27.
22. 1050; 950.
23. 8.
24. 51; 3.
25. 50.
26. 6000.
27. 21 inches; 15 inches.
28. 12; 11.
29. 27; 36.
30. Sheep, \$6; cow, \$42; horse, \$168.
31. Henry, \$3; Thomas, \$6; Richard, \$6.
32. A, \$100; B, \$400; C, \$500.
33. James, 4 years; John, 12 years.
34. 16.
35. 23.
36. 3; 12.
37. Cloth, \$1; silk, \$2.
38. 18 children; 54 women; 108 men.
39. 6.
40. \$240.
41. Left standing, 14 feet; broken off, 70 feet.
42. 1480; 1320.
43. 11; 9.
44. 16; 34.
45. 7; 12.
46. 18.
47. 4.
48. 61.
49. 4; 15.
50. Son, 30 years; father, 60 years.

51. Son, 10 years; father, 40 years. 54. 22. 55. 24.
 52. Son, 14 years; father, 35 years. 56. 2 miles an hour. 57. \$11.
 53. 18 miles. 58. A, \$3; B, \$5; C, \$7.
 59. 14. 60. 11.
 61. A, \$7 $\frac{1}{2}$; B, \$2 $\frac{3}{4}$; C, \$1 $\frac{1}{2}$.
 62. Son, 26 years; father, 54 years. 63. 42 gallons.
 64. 60 pounds at \$0.50; 40 pounds at \$0.75. 65. \$2.
 66. 4 dozen at \$0.25; 5 dozen at \$0.30.
 67. 5 quarters and 10 ten-cent pieces.
 68. 6 ten-cent pieces and 24 five-cent pieces.
 69. 16 quarters and 4 half-dollars.
 70. 4 quarters and 20 half-dollars.
 71. 5 ten-dollar bills; 15 one-dollar bills.
 72. A, \$2.50; B, \$3.50; C, \$4.00.
 73. 3 silver dollars; 9 half-dollars; 15 quarters.
 74. Calico, 6 cents; cloth, \$1.50.
 75. 5 dollars; 10 half-dollars; 20 quarters.

Exercise 9. Page 37.

- | | | | | | |
|---------|---------|----------|----------|----------|-----------|
| 1. +1. | 5. +7. | 9. -22. | 13. +7. | 17. +2. | 21. +1. |
| 2. -1. | 6. -15. | 10. +40. | 14. +3. | 18. -29. | 22. -3. |
| 3. -14. | 7. -11. | 11. +9. | 15. -22. | 19. +20. | 23. -215. |
| 4. -7. | 8. -6. | 12. -17. | 16. -10. | 20. -42. | 24. +50. |

Exercise 10. Page 38.

- | | | | |
|-----------|-----------|------------------------|---|
| 1. 2 a. | 4. -6 ab. | 7. -4 z ⁴ . | 10. -17 y ⁴ z ⁴ . |
| 2. 10 xy. | 5. 16 c. | 8. -7 m ² . | 11. 0. |
| 3. -yz. | 6. 0. | 9. -2 abc. | 12. -19 y ⁵ . |

Exercise 11. Page 39.

- | | | | | |
|-------------------------|--------------------------|---------------------------|--|--------------------------|
| 1. a. | 8. ac. | 15. -34 c ² . | 22. 15 abcx. | 28. - $\frac{1}{3}$ xy. |
| 2. 0. | 9. 21 yz. | 16. -9 m ² . | 23. 10 y ⁴ z ⁴ . | 29. - $\frac{5}{8}$ z. |
| 3. -4 a ² b. | 10. -12 ab. | 17. x ² . | 24. -3 x ⁵ y ⁵ . | 30. $\frac{27}{4}$ ayz. |
| 4. 11 xy. | 11. 4 x ² . | 18. -23 a ² . | 25. abz ³ . | 31. a. |
| 5. 6 b. | 12. 13 y ³ . | 19. -22 a ³ x. | 26. $\frac{4}{3}$ a. | 32. -8 a ² c. |
| 6. 3 c. | 13. -12 z ⁴ . | 20. 0. | 27. $\frac{17}{4}$ a ² . | 33. 5 x ² y. |
| 7. xz. | 14. -3 y ² . | 21. -15 ax ³ . | | |

Exercise 12. Page 41.

- | | | | | | |
|---------|--------|--------|---------|----------|---------|
| 1. 8. | 4. 14. | 7. -5. | 10. 12. | 13. -15. | 16. -1. |
| 2. -8. | 5. -8. | 8. 11. | 11. 6. | 14. -15. | 17. 10. |
| 3. -14. | 6. 8. | 9. 5. | 12. -6. | 15. 15. | 18. 10. |

Exercise 13. Page 41.

- | | | | |
|-------------|---------------|-----------------|------------|
| 1. $27b$. | 5. $-10m^2$. | 9. $4abc$. | 13. 9. |
| 2. $12yz$. | 6. $70c$. | 10. $-4ac^2x$. | 14. 3. |
| 3. $-3ab$. | 7. $-70c$. | 11. 3. | 15. -9 . |
| 4. $4a^2$. | 8. $12c$. | 12. -9 . | 16. 5. |

Exercise 14. Page 45.

- | | | | |
|---------------|--------------------|-------------------|---------------------|
| 1. $6a^2$. | 9. $-56a^6$. | 17. a^4b^5 . | 25. $30abx^5y^5$. |
| 2. $6a^2$. | 10. $-56m^6$. | 18. $54a^{m+n}$. | 26. $-48a^5b^5$. |
| 3. $-6a^2$. | 11. $-5x^4y^4$. | 19. a^{2m} . | 27. $-3a^8b^8c^8$. |
| 4. $-6a^2$. | 12. $-a^4b^4c^5$. | 20. $-2x^n$. | 28. $3x^5y^5z^5$. |
| 5. $54a^8$. | 13. $9a^8x^3$. | 21. z^{2n+1} . | 29. $1000a^7b^7$. |
| 6. $-54a^4$. | 14. $30a^8c^4$. | 22. a^{n+1} . | 30. $48a^4b^7$. |
| 7. $35x^7$. | 15. $-21x^{12}$. | 23. y^{2n+2} . | 31. a^{5n+2} . |
| 8. $7a^5$. | 16. $54a^2b^2$. | 24. x^n . | 32. x^{5m-1} . |

Exercise 15. Page 48.

- | | | | |
|---------------|------------------------|--------------------------|-----------------|
| 1. $2b$. | 11. $9ac^2$. | 21. a^2x^7 . | 31. $3cy$. |
| 2. $2ac^2$. | 12. $-9bx$. | 22. $-3x^2y$. | 32. $2a^2b^2$. |
| 3. $4y^2$. | 13. $-3x^2y^2z$. | 23. $-4cd^2$. | 33. ab^4 . |
| 4. $2x^2y$. | 14. $2c$. | 24. $-4x^2$. | 34. z . |
| 5. $9axc^3$. | 15. $\frac{2}{3}x^2$. | 25. $-4x^2yz$. | 35. y^2z^5 . |
| 6. $-4cy^2$. | 16. $-8a^2y^2$. | 26. ac . | 36. $2c^3y$. |
| 7. $3ax^2$. | 17. $3b^2$. | 27. $-3a^2$. | 37. $2a^2$. |
| 8. $7ax^8$. | 18. $-2abz^2$. | 28. $2x^{n-2}y^{n-1}$. | 38. $2a^8$. |
| 9. $-9yz$. | 19. $-2x$. | 29. $-6x^{n-4}y^{n-3}$. | 39. $-3xy$. |
| 10. $6xz^2$. | 20. $-3a^3$. | 30. $x^{5-n}y^{4-n}$. | 40. xz^2 . |

Exercise 16. Page 51.

- | | |
|--|------------------------------------|
| 1. $2a$. | 6. $9x + 2y - z$. |
| 2. $x^3 - x$. | 7. $3ax^2 - 7a^2x + 2x^3$. |
| 3. $8x^2 - x$. | 8. $4a^4 - 7a^3 + 7a^2 - 2x + 8$. |
| 4. $4x^2 - 4xy + 4y^2$. | 9. $4a^2 - 2b^2 + 6c^2$. |
| 5. $-3bx + 5$. | 10. $10x^4 + x^3 + x^2 - x$. |
| 11. $6y^3 - 13xy^2 - 3x^2y + 6x^3 - 17xz^2 - z^3$. | |
| 12. $2a^4 + 4a^3 + 5a^2 - a - 4$. | |
| 13. $4x^3 - 4x^2y - 10xy^2 - 15y^3$. | |
| 14. $-c^4 + c^3 + c + 8$. | |
| 15. $4x^2 - 2xy - xz - 2y^2 + 2z^2 + 12yz - 6y - 6z$. | |
| 16. $2m^5 - 5m^4n - 5m^3n^2 + 3m^2n^3 - 4mn^4 - n^5$. | |
| 17. $7xy^2 - 6x^2y + 2$. | |

Exercise 17. Page 53.

- | | |
|---|-----------------------------|
| 1. $6a - b + c.$ | 8. $a^2 - 7ab + b^2.$ |
| 2. $a + 4b + 4c + d.$ | 9. $4a^2b + 10ab^2 + 1.$ |
| 3. $2a^2 - 3x + 2.$ | 10. $10a^2c - 8a^3 - b^4.$ |
| 4. $x^2 - ax + 2a^2.$ | 11. $a^2 - 2b^2.$ |
| 5. $2a - 7c.$ | 12. $-b^2.$ |
| 6. $2x^2 + 14x + 2.$ | 13. $-a^2 + 2b^2.$ |
| 7. $-ax + 6by + 10.$ | 14. $b^2.$ |
| 15. $-2x^4 + 2ax^3 + 2bx^2 - 3cx - 5d.$ | |
| 16. $8a^2 - 12ab + 17b^2.$ | 19. $-10a^2 - 2ab + 11b^2.$ |
| 17. $-2a^2 + 8ab - 7b^2.$ | 20. $16a^2 - 2ab - b^2.$ |
| 18. $6a^2 - 4ab - 7b^2.$ | 21. $12a^2 - 8ab + 3b^2.$ |

Exercise 18. Page 55.

- | | | | |
|--------------------------------|---------------|--------------------|----------------|
| 1. 0. | 4. $5c - 3b.$ | 7. $3x - 3y.$ | 10. $a.$ |
| 2. $p - n.$ | 5. $d - c.$ | 8. $-3c.$ | 11. $a + 5c.$ |
| 3. $4x - 4z.$ | 6. $2x - 2.$ | 9. $2a.$ | 12. $-x + 2y.$ |
| 13. $-11a.$ | | 18. $4a - 2c - b.$ | |
| 14. $2x - 3y - z - b + a - c.$ | | 19. $-2a + b + c.$ | |
| 15. $5a - b - c.$ | | 20. $8a - b.$ | |
| 16. $15 - x.$ | | 21. $23 - 15a.$ | |
| 17. $-2x - 2y + 6z.$ | | 22. $x - 6y + 4z.$ | |

Exercise 19. Page 56.

- $2a - b + 3c - (d - 3e + 5f).$
- $x - a - y - (b + z + c).$
- $a + b - c - (-4a + b - 1).$
- $ax + by + cz - (-bx + cy - cz).$
- $3a + 2b + 2c - (5d + 3e + 4f).$
- $x - y + z - (5xy + 4xz - 3yz).$
- $(a + b)x + (b - c)y + (a + c)z.$
- $(a - b)x + (2a + 3)y + (4a - 3b - 2)z.$
- $(a - 4b)x - (2b - 3c)y - (7a + 5c)z.$
- $(a - 11c)x + (3a + 2b + 2c)y - (b + c)z.$
- $(2b - 3a - 7c - 1)x + (4b - 5c - 1)y - (6c + 1)z.$
- $-x - (3a + 5b - 1)y + (6a - 2b + 3c + 1)z.$
- $(2a - 2m)x - (b + 3c)y + (3a - 5b + 1)z.$
- $(1 - ac)x - (1 - a + mn)y - (1 + a - bc)z.$
- $(2a - 4b - 2c)x - (6a + 3c)y + 4bz.$
- $(a - b)x + (2a + 3)y + (4a - 3b - 2)z.$
- $(a - 4b - 2c)x - (a + 2b + 3c)y + (a + 4b + 5c)z.$
- $(12a - 15c)x + (12a + 4b + 6c)y - (12b - 3c)z.$

Exercise 20. Page 57.

1. $17x^3 - 2a^2x - 5a^2 + 11a^3 - 11x^2$.
2. $9ab + 7a + 14b + 5b^2 + 2a^2$.
3. $-6(5 - x)$.
4. $2(a + b + c)(x^2 + y^2 + z^2)$.
5. $2ax + 2cy + 2(a + b + c)z$.
6. $-2x^2 - 2ax$.
7. $2a^2 + 3ax + 2x^2$.
8. $3x^2 - 5ax$.
9. $a^3 + 3b^2c - b^3$.
10. $2bx + 2ay$.
11. $5a$.
12. $2a - b - d$.
13. $(a^2 + b - 1)x^3 - (c^2 + c - 1)x^2 - (a + b + c)x$.
14. $(8a - b - 7c)x^3 - (a + 3b - 2c)x^2 + (2a - 2b - 2c)x$.
15. 49.
16. 42.
17. $2d + 6a + 13b - 7c$.
18. 94.
19. -160.
20. 106.
21. 53.
22. $7a^3 - 5a^2b - 5b^3$.
23. $3a^3 + 11a^2b + b^3$.
24. $-x^2 - 3y^2 - 4z^2$.
25. $-4a^3 + 5a^2b - 5ab^2 + b^3$.
26. $11a^3 - 7a^2b + b^3$.
27. $a^3 + 9a^2b - 3b^3$.
28. $-a^3 + 13a^2b - 5b^3$.
29. $a^3 - 13a^2b + 5b^3$.
30. $-11a^3 + 7a^2b - b^3$.
31. $-3a^3 - 11a^2b - b^3$.
32. $x^2 - 3y^2 - 3z^2$.
33. $2a^3 - 2a^2c + 2ac^2$.

Exercise 21. Page 60.

1. $10a^3 + 6a^2b$.
2. $5a^4b^2c - 5a^3b^2c^2$.
3. $a^2b^2c - a^2bc^2 - ab^2c^2$.
4. $6a^7b^3c - 7a^4b^4c^2$.
5. $-3x^6y^7z^9 + 3x^5y^8z^9 + 3x^5y^7z^{10}$.
6. $-3x^4 - 6x^2y^2 + 3x^2z$.
7. $a^5bc^2 + a^3b^3c^2 - a^3bc^4$.
8. $20a^3b^3c^2 - 12ab^5c^2 + 8ab^3c^4$.
9. $-2a^2b^3c^2 + 6a^4b^3c^3$.
10. $-x^3y^2z^3 - x^4y^4z^2$.
11. $15x^3 - 10x^2y - 20x^2$.
12. $6x^4y - 8x^2y^3 + 10x^2yz^2$.
13. $a^4x^3y - 5a^3x^4y + a^2x^5y + 2ax^6y$.
14. $27a^6b^4 - 9a^4b^6 + 12a^3b^7 + 3ab^9$.
15. $-15x^5y + 10x^4y^2 + 35x^3y^3 - 5x^2y^4$.
16. $12x^3y^3 - 15x^4y^2 - 24x^5y$.
17. $3a^4 - 2a^5b - a^6b^2$.
18. $3x^3yz^2 + 6x^4yz^3 - 15x^5y^2z^3$.

Exercise 22. Page 63.

1. $x^2 + 16x + 60$.
2. $x^2 - 5x + 6$.
3. $x^2 + 2x - 15$.
4. $x^2 - 9$.
5. $x^2 - 12x + 11$.
6. $x^2 + x - 6$.
7. $-x^2 + 4$.
8. $-x^2 + 8x - 16$.
9. $-x^2 + 49$.
10. $x^2 - 49$.
11. $2x^2 - 3x - 9$.
12. $2x^2 + 3x - 9$.
13. $2x^2 - 15x + 7$.
14. $2m^2 - 2mn + m - n$.
15. $m^2 - a^2$.
16. $6x^2 + 5x - 21$.

17. $25x^2 - 4y^2$.
 18. $6x^2 + xy - 12y^2$.
 19. $x^5 + x^3y^2 - x^2y^3 - y^5$.
 20. $2x^4 + 5x^2y^2 + 3y^4$.
 21. $x^2 + 2xz - y^2 + z^2$.
 27. $a^4 - 9a^3 + 19a^2 - 25a + 6$.
 28. $6x^4 - x^3y - 10x^2y^2 + 31xy^3 - 20y^4$.
 29. $x^4 + x^3y + x^2y^2 - x^3z - x^2yz - xy^2z - x^2z^2 - xyz^2 - y^2z^2$.
 30. $x^3 - 3xyz + y^3 + z^3$.
 31. $-50ab^2 + 125a^2b^3 + 8a^3 + 50a^3b^3$.
 32. $x^2y^2 + 4y^3 + 4x^3 + 16xy$.
 33. $5x^2y^2 + 2xy^3 + 8y^2 + 2x^3y - 8x^2 - 64$.
 34. $a^3 - 3ab + 1 + b^3$.
 35. $24x^4 - 10x^2y^2 + 31x^2z^2 - 4y^4 + 16y^2z^2 - 15z^4$.
 36. $a^3 - 3abc + b^3 + c^3$.
 38. $x^5 + 151x - 264$.
 37. $x^3 + 3xy + y^3 - 1$.
 39. $x^5 - 41x - 120$.
 40. $x^5 + 10x - 33$.
 41. $x^7 - 7x^5 + 21x^3 - 17x - 25x^3 + 4x^2 + 2x$.
 42. $4x^5 - 5x^5 + 8x^4 - 10x^3 - 8x^2 - 5x - 4$.
 43. $25a^7b - 9a^5b^3 + 22a^4b^4 - 4a^3b^5 + ab^7$.
 44. $4a^{13}y^3 + 8a^{11}y^4 - 64a^5y^7 - 128a^3y^8$.
 45. $24a^3b - 26a^5b^5 + 4a^4b^6 - 6a^3b^7 - 5a^2b^8 + 4ab^9 - 3b^{10}$.
 46. $x^3 + 3xy^2 + 3x^2y - 3x^2 - 6xy + x + y^3 - 3y^2 + y + 1$.
 47. $a^{m+1} + 3a^m - a^{m-1} - 3a^{m-2} - a - 1$.
 48. $a^{n+1} - 5a^n + 9a^{n-1} - 4a^{n-2} - a^{n-3}$.
 49. $2a^{4n+4} - 8a^{3n+3} + 4a^{2n+2} - 2a^{n+1} - a^{4n+3} + 4a^{3n+2} - 2a^{2n+1} + a^n$
 $+ a^{4n+2} - 4a^{3n+1} + 2a^{2n} - a^{n-1}$.
 50. $x^{2n} - y^{2n+2}$.
 51. $x^{4n} - 2x^{2n}y^{2n} + y^{4n}$.
 52. $a^{2n} - a^{2n+1} - a^{2n+2} + 2a^n - a^{n+1} - a^{n+2} + 1$.
 53. $a^{4n} - 2a^{3n} + 2a^{2n} - 2a^n + 1$.
 54. $a^{n+2} - a^{n+1} + 2a^{n-1} - a^{n-2} - a^{n-3}$.

Exercise 23. Page 64.

1. $x^4 + x^3 - 24x^2 - 35x + 57$.
 2. $-x^5 - 3x^2 - x + 1$.
 3. $4x^5 - x^3 + 4x$.
 4. $18x^4 - 45x^3 + 82x^2 - 67x + 40$.
 5. $x - y - x^2y + xy^2 + x^2 + xy - y^2$.
 6. $-70x^5 + 106x^4 - 112x^3 + 82x^2 - 38x + 12$.
 7. $32a^5b - 56a^4b^2 + 8a^3b^3 - 4a^2b^4 - ab^5$.
 8. $-x^3 - x^2 + ax + 19x - b - 12$.
 9. $x^5 + (c-m)x^4 + (d-cm+n)x^3 + (cn-dm+r)x^2 + (cr+dn)x + dr$.
 10. $x^3 - (a+b+c)x^2 + (ab+ac+bc)x - abc$.
 11. $x^4 - y^4$.
 12. $16x^4 + 36x^2y^2 + 81y^4$.
 13. $x^7 - 2x^6 + x^5 - 2x^4 - 7x^3 + 14x^2 + 20x - 40$.
 14. $x^3 + x^4y^4 + y^8$.

15. $2x^8 - 6x^7 - 19x^6 + 61x^5 + 12x^4 - 109x^3 + 111x^2 + 40x - 120$.
 16. $a^{3m+1} + a^{3m}y + ay^{3m} + y^{3m+1}$. 17. $a^{5x} - a^{4x} - a^x + 1$.
 18. $a^3 + 3a^2b + 3ab^2 + b^3 + c^3$. 19. $-a^2 + 10b^2$.
 20. $a^2 + b^2$. 21. $2mn$. 22. 0. 23. $-6x$.
 24. $24ab - 120b^2$. 26. $-3bx + 2ay$.
 25. $2x^2 + 2y^2 + 2z^2$. 27. $ax - cx - ab + ac + bc - c^2$.
 28. $b^2 - d^2$. 30. $a^2 + b^2$. 32. 0. 34. 0.
 29. 0. 31. 6. 33. 0. 35. 17; 13.

Exercise 24. Page 66.

1. $x + 2y$. 7. $3a - 2a^3$. 12. $-a^2 + 2ab + 3b^2$.
 2. $a - 2b$. 8. $3x^4y^2 + 1$. 13. $x^3 - xy - 1$.
 3. $2x^3 - 4$. 9. $x^4z^4 + 2x^2z^2$. 14. $a^2b - ab^2 - ac^2$.
 4. $3x^2 + 1$. 10. $c^2 - 3a^3b^4$. 15. $-4a^2 + 2ab + 3b^2$.
 5. $a^2 + 2a^7$. 11. $-x + y + z$. 16. $m^2 - 2mn - 3n^2$.
 6. $3a - 4$.

Exercise 25. Page 70.

1. $a + 3$. 4. $x - y$. 7. $2x - 1$. 10. $-a^2 - 3a - 1$.
 2. $a - 2$. 5. $x + y$. 8. $2x - 3a$. 11. $a^2 - 2a + 2$.
 3. $x + y$. 6. $2x + 3$. 9. $-3a - 2$. 12. $y^2 - 3y - 1$.
 13. $m^2 - 5m + 6$. 29. $x - 2y + 3z$.
 14. $1 - 3s + 2s^2 - s^3$. 30. $x^3 - 4x^2 + 11x - 24$.
 15. $b^4 + 2b^3 + 3b^2 + 2b + 1$. 31. $-x^2 + x - 1$.
 16. $x^2 + 2xy + 3y^2$. 32. $x^2 - 3x - 2$.
 17. $a + b$. 33. $5x^4 + 4x^3 + 3x^2 + 2x + 1$.
 18. $1 + x - 2x^2$. 34. $-x^2 - 3x - 9$.
 19. $4x^2 + 4xy + 3y^2$. 35. $x - y$.
 20. $x^2 + 2xy - xz + y^2 - yz + z^2$. 36. $x^4 - x^2y^2 + y^4$.
 21. $a^2 - ab - ac + b^2 - bc + c^2$. 37. $x^2 + ax + a^2$.
 22. $x + 2y + z$. 38. $a + 2b - c$.
 23. $x - y - z$. 39. $a - b - c$.
 24. $x - y - z$. 40. $5a^2 + 3x^2$.
 25. $x^2 - xy + y^2$. 41. $a^2 + 2ab + a + 4b^2 - 2b + 1$.
 26. $x^2 - 3x + 2$. 42. $x^{2n} - 2x^ny^n + y^{2n}$.
 27. $y^2 - 5y + 6$. 43. $a^n b^n - 7a^{n-1}b^{2n}$.
 28. $y^2 + yz - 2z^2$.

Exercise 26. Page 73.

1. $\frac{1}{3}a^2b + \frac{2}{15}b^2c^4 - \frac{1}{4}z$. 3. $\frac{1}{3}y - \frac{2}{4}a - \frac{1}{12}x + \frac{1}{3}b$.
 2. $\frac{1}{2}x^2 + \frac{1}{2}ax - \frac{1}{6}a^2$. 4. $\frac{1}{3}c^4 - \frac{1}{6}c^3 + \frac{1}{16}c^2 - \frac{1}{4}$.

5. $\frac{1}{4}x^2 + \frac{5}{8}x^4 + \frac{1}{16}x^6$.
 6. $0.2m^6 - 0.46m^5n + 0.32m^4n^2 - 0.08m^3n^3 - 2m^2n^4 + 0.2mn^5 + 1.12n^6$.
 7. $\frac{3}{4}a^3 - \frac{3}{4}a^2b + \frac{1}{4}ab^2$.
 8. $\frac{1}{4}d^3 - \frac{3}{4}d^2 + \frac{1}{2}d$.

Exercise 27. Page 73.

1. 0. 2. -11 ; 11. 4. $a^{3m} + b^{3m}$.
 3. $a^3 + \frac{1}{2}a^2b + \frac{1}{4}b^3$. 5. $8a^{3m+3} - 27a^6$.
 6. $x^3 - 2xy + 5xz + 4y^2 + 10yz + 25z^2$.
 7. $ax - bx$. 14. $y^{m-p-1} - y^{-3m-2p+2} + y^{m+2p}$.
 8. $1 - 2b$. 15. $2x^{2n} - 4x^ny^n + 2y^{2n}$.
 9. $20x - 35x^{m-5n+4} + 25x^{2m}$. 16. $x - a - b$.
 10. $c^{m-1}d - c^{m+1}d^2 - c^{2m+1}d^{1+x}$. 17. $x^{2m} + x^m + 1$.
 11. $m^{x-2}y^4 - m^{2x-4}y^5 + 1$. 18. $3x^3 + 5x^2 + 3x$.
 12. $a^{x+y-1} - a^{x-2y+1} + a^{x+3y}$. 19. $3x^{2m+5} - 2x^{m+5} + 5x^5$.
 13. $x^{m+2n-1} - x^{2m-3n+2} + x^{3m+2n-2}$. 20. $3a^{3n-3} + 2a^{2n-2} - 5a^{n-1}$.
 21. $(1 - a^2)x^3 - (a^2 + a)x^2 - (a + 2b + bc)x$.
 22. $2a^3 - 2.49a^2x - 0.3ax^2 + 0.5x^3$. 25. $\frac{1}{2}a^2 - \frac{1}{3}ab + \frac{1}{4}b^2$.
 23. $\frac{1}{4}a^3 + \frac{1}{15}ab^2 + \frac{1}{15}b^3$. 26. $\frac{1}{12}x^2 - \frac{7}{16}xy - \frac{1}{24}y^2$.
 24. $\frac{1}{8}a^3 + \frac{1}{18}a^2b + \frac{1}{18}ab^2 - \frac{1}{2}b^3$. 27. $x^2 - \frac{5}{8}xy + \frac{3}{8}y^2$. 28. 14.

Exercise 28. Page 76.

1. $x^2 + 2xy + y^2$. 7. $x^2 - y^2$.
 2. $x^3 - 2ax + a^2$. 8. $16z^2 - 9$.
 3. $x^2 + 4bx + 4b^2$. 9. $9a^4 - 16b^4$.
 4. $9x^2 - 12cx + 4c^2$. 10. $9a^2 - 6ac + c^2$.
 5. $16y^2 - 40y + 25$. 11. $x^2 + 14b^2x + 49b^4$.
 6. $9a^4 + 24a^2z^2 + 16z^4$. 12. $a^2x^2 - 4b^2y^2$.

Exercise 29. Page 77.

1. $x^2 - y^2 - 2yz - z^2$. 9. $a^4 + a^2 + 1$.
 2. $x^2 - 2xy + y^2 - z^2$. 10. $9x^2 - 4y^2 + 4yz - z^2$.
 3. $a^2x^2 + 2abxy + b^2y^2 - 1$. 11. $1 + 2x + x^2 - y^2$.
 4. $1 - x^2 + 2xy - y^2$. 12. $a^4 + 4a^2x^2 + 16x^4$.
 5. $a^2 - 4b^2 + 12bc - 9c^2$. 13. $x^4 - 2x^2y^2 + y^4$.
 6. $a^4 + a^2b^2 + b^4$. 14. $x^2 - 2xy + y^2 - 169z^8$.
 7. $m^4 + m^2n^2 + n^4$. 15. $x^4 - 10x^2y^2 + 25y^4 - 49z^8$.
 8. $4 - x^2 - 2x^3 - x^4$.

Exercise 30. Page 78.

1. $4x^2 - 12xy + 9y^2$.
2. $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.
3. $x^2 + y^2 + z^2 + 2xy - 2xz - 2yz$.
4. $x^2 + y^2 + z^2 - 2xy + 2xz - 2yz$.
5. $x^2 + y^2 + 2xy + 10x + 10y + 25$.
6. $x^2 + 4y^2 + 4xy + 6x + 12y + 9$.
7. $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$.
8. $9x^2 + 4y^2 - 12xy + 2x - 16y + 16$.
9. $4x^2 + 9y^2 + 16z^2 - 12xy + 16xz - 24yz$.
10. $x^4 + y^4 + z^4 + 2x^2y^2 + 2x^2z^2 + 2y^2z^2$.
11. $4x^2 + y^2 + z^2 - 4xy - 4xz + 2yz$.
12. $a^2 + 4b^2 + 9c^2 - 4ab - 6ac + 12bc$.
13. $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$.
14. $x^2 + 4y^2 + 9z^2 + 4xy - 6xz - 12yz$.
15. $x^4 + y^4 + z^4 - 2x^2y^2 + 2x^2z^2 - 2y^2z^2$.
16. $x^2 + 4y^2 + 9z^2 - 4xy - 6xz + 12yz$.
17. $x^2 + y^2 + 4z^2 + 2xy - 4xz - 4yz$.
18. $x^2 + y^2 + z^2 + 1 + 2xy + 2xz + 2x + 2yz + 2y + 2z$.
19. $x^2 + y^2 + z^2 + 1 - 2xy + 2xz - 2x - 2yz + 2y - 2z$.
20. $16x^2 + y^2 + z^2 + 8xy + 8xz - 16x + 2yz - 4y - 4z + 4$.
21. $4x^2 + y^2 + z^2 - 4xy - 4xz - 12x + 2yz + 6y + 6z + 9$.
22. $x^2 + 4y^2 + 9z^2 - 4xy - 6xz + 8x + 12yz - 16y - 24z + 16$.

Exercise 31. Page 80.

1. $x^2 + 11x + 24$.
2. $x^2 + 5x - 24$.
3. $x^2 + 3x - 70$.
4. $x^2 - 14x + 45$.
5. $x^2 - x - 90$.
6. $a^2 - 15a + 50$.
7. $x^2 - ax - 6a^2$.
8. $a^2 - 2ab - 8b^2$.
9. $a^2 - 15a + 36$.
10. $a^2 + 6ab + 8b^2$.
11. $a^2 + 4ab - 21b^2$.
12. $a^2 - 7ab - 18b^2$.
13. $x^2 - 7ax + 12a^2$.
14. $x^2 + 2xz - 8z^2$.
15. $x^2 + xy - 30y^2$.
16. $x^4 - x^2 - 72$.
17. $x^4 - x^2y^2 - 6y^4$.
18. $x^4 + 4x^2y^2 - 32y^4$.
19. $a^2b^2 - 3ab - 40$.
20. $a^2b^2 - 4abxy - 21x^2y^2$.
21. $x^2 - 6xy + 9y^2$.
22. $x^2 + 12x + 36$.
23. $a^2 - 6ab + 9b^2$.
24. $x^2 - (c + d)x + cd$.
25. $x^2 + (a - b)x - ab$.
26. $x^2 - (a - b)x - ab$.
27. $(a + b)^2 - 2(a + b) - 8$.
28. $(x + y)^2 + 2(x + y) - 8$.

Exercise 32. Page 81.

1. $6x^2 + xy - y^2$.
2. $12x^2 - 17xy + 6y^2$.
3. $15x^2 - 32xy + 16y^2$.
4. $2x^2 - 19xy + 35y^2$.

- | | |
|------------------------------------|------------------------------|
| 5. $77x^2 - 3xy - 2y^2$. | 13. $12a^4 + a^2x^2 - x^4$. |
| 6. $100x^2 - 100xy + 21y^2$. | 14. $2x^2 + 5xy + 2y^2$. |
| 7. $6a^4 + 5a^2b^2 - 6b^4$. | 15. $6b^2 - 7bx + 3x^2$. |
| 8. $a^3 - a^2b + ab^2 - b^3$. | 16. $8a^2 + 14ab - 15b^2$. |
| 9. $6a^3 + 9a^2b - 4ab^2 - 6b^3$. | 17. $8x^2 - 22xy - 21y^2$. |
| 10. $a^3 + a^2b - ab^2 - b^3$. | 18. $6y^2 + 7yz - 3z^2$. |
| 11. $12x^2 - 5x - 2$. | 19. $6x^2 + 19xy - 7y^2$. |
| 12. $3x^2 - 2x - 5$. | 20. $6a^2 - 19ac + 10c^2$. |

Exercise 33. Page 82.

- | | | | |
|--------------|---------------|--------------------|-----------------------|
| 1. $a + 2$. | 4. $x + 5$. | 7. $3a^2 - 5y^2$. | 10. $ab^3c^4 - x^5$. |
| 2. $3 - x$. | 5. $6 - x$. | 8. $2x^3 + 3y^3$. | 11. $x^2a^4 + b^5$. |
| 3. $4 - a$. | 6. $3a + b$. | 9. $2x^5 + a^4$. | 12. $a + b + c$. |

Exercise 34. Page 83.

- | | | |
|----------------------------|----------------------------|------------------------------|
| 1. $1 + 2x + 4x^2$. | 7. $1 + 3z + 9z^2$. | 12. $49 + 14a + 4a^2$. |
| 2. $1 - 2x + 4x^2$. | 8. $1 - 3z + 9z^2$. | 13. $4a^2 - 2ab^2 + b^4$. |
| 3. $9a^2 + 3ab + b^2$. | 9. $a^2b^2 + abc + c^2$. | 14. $x^4 - 9x^2y + 81y^2$. |
| 4. $9a^2 - 3ab + b^2$. | 10. $a^2b^2 - abc + c^2$. | 15. $a^4 + 3a^2b + 9b^2$. |
| 5. $16x^2 - 12xy + 9y^2$. | 11. $16 - 4y + y^2$. | 16. $4x^2 + 8xy^2 + 16y^4$. |
| 6. $16x^2 + 12xy + 9y^2$. | | |

Exercise 35. Page 84.

- | | |
|--|------------------------------------|
| 1. $x^5 + x^4y + x^3y^2 + x^2y^3 + xy^4 + y^5$. | 6. $x^4 + 2x^3 + 4x^2 + 8x + 16$. |
| 2. $x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5$. | 7. $x^4 - 2x^3 + 4x^2 - 8x + 16$. |
| 3. $x^3 + x^2 + x + 1$. | 8. $1 + m + m^2 + m^3$. |
| 4. $x^3 - x^2 + x - 1$. | 9. $1 - m + m^2 - m^3 + m^4$. |
| 5. $x^3 + 2x^2 + 4x + 8$. | |

Exercise 36. Page 86.

- | | |
|-----------------------------|-----------------------------------|
| 1. $3x^2(1 - 2x)$. | 13. $4a^2(2x^2 - b + 3y^2)$. |
| 2. $2a(a - 2)$. | 14. $2a^2b^2c^2(4a - 2bc + c)$. |
| 3. $5ab(1 - ab^2)$. | 15. $5a^3(3x - 2y + z)$. |
| 4. $ab(3a - 4b)$. | 16. $a^2cy^2(ay + 2c - y^2)$. |
| 5. $4x^2y^2(2x + y)$. | 17. $bc^2(3b^2c + 2b - 6c)$. |
| 6. $a(3a^2 - a + 1)$. | 18. $3ab(2a - 1 - 4b)$. |
| 7. $x(x^2 + xy - y^2)$. | 19. $a^2c(5c^2 + 3 + 4c)$. |
| 8. $a^2(a^2 - ab + b^2)$. | 20. $3xy^2(2x^2 + x - 5y)$. |
| 9. $3x^2(x^2 - 3 - 2x)$. | 21. $7abc(abc - 2 + bc^2)$. |
| 10. $b(ab - c^2 + x)$. | 22. $8xy(xy + 2z - 3xyz^2)$. |
| 11. $2a(4ab - 3a^2 + 2b)$. | 23. $abc(bc^2 - 2a + 3a^2b^2c)$. |
| 12. $4xy(x - 2y - 1)$. | 24. $x^2y^2z(z - xz^2 + y)$. |

Exercise 37. Page 87.

1. $(x + y)(a - b)$.
2. $(x - y)(a - b)$.
3. $(x - y)(a + c)$.
4. $(x - b)(x + a)$.
5. $(x - a)(x + y)$.
6. $(x - b)(x - y)$.
7. $(a - y)(2b - 3c)$.
8. $(x + 2a)(2x - 3y)$.
9. $(b - 2c)(a - 3c)$.
10. $(x - z)(x^2 + 1)$.
11. $(x^2 + 3)(x + 4)$.
12. $(3a - 1)(c - x)$.
13. $(ab - c)(a - x)$.
14. $(x^2 - 2)(2x - 3)$.
15. $(x^3 - 1)(ax + b)$.
16. $(ax + 1)(a + x)$.
17. $(x - y)(x + y)$.
18. $(1 - 3x^3)(1 - 5x)$.
19. $(x^2 + 1)(1 - x)$.
20. $(x^2 + 2)(x - 5)$.
21. $(x^2 + 3)(x + 7)$.
22. $(z^3 - 1)(z - 1)$.
23. $(bx - cy)(a^2 + bc)$.
24. $(3x^2 - 5y^3)(1 - 2x)$.
25. $(2x - 3)(4a - 5b)$.
26. $x(2x^2 - 3)(3x + 4)$.
27. $x(x^2 - 3)(3cx - 2d)$.
28. $(a + b)(d - 2c)$.
29. $(b^2 + c^2)(bx - cy)$.
30. $(2a + c)(2a - b)$.
31. $(an + c^2)(a - m)$.
32. $(1 + c)(1 - c^2xy)$.

Exercise 38. Page 89.

1. $(a - 3b)(a - 3b)$.
2. $(2a + b)(2a + b)$.
3. $(a - 2b)(a - 2b)$.
4. $(x + 3y)(x + 3y)$.
5. $(2x - 3a)(2x - 3a)$.
6. $(a - 5b)(a - 5b)$.
7. $(2a - 1)(2a - 1)$.
8. $(7y - z)(7y - z)$.
9. $(x - 8)(x - 8)$.
10. $(3x + 4y)(3x + 4y)$.
11. $(4a + x)(4a + x)$.
12. $(5 + 8x)(5 + 8x)$.
13. $(7x - 2y)(7x - 2y)$.
14. $(1 - 10b)(1 - 10b)$.
15. $(9a + 7b)(9a + 7b)$.
16. $(mn - 8a^2)(mn - 8a^2)$.
17. $(2a - 5x)(2a - 5x)$.
18. $(11a + 9y)(11a + 9y)$.
19. $(ab^2c^3 - x^8)(ab^2c^3 - x^8)$.
20. $(7 - 10k^2)(7 - 10k^2)$.
21. $(7a + 3c^2)(7a + 3c^2)$.
22. $(9c - 5)(9c - 5)$.
23. $(11 + 5x)(11 + 5x)$.
24. $(12 + 7z)(12 + 7z)$.
25. $(6x - 5y)(6x - 5y)$.
26. $(y - 25z)(y - 25z)$.
27. $(x^3 - 17)(x^3 - 17)$.
28. $(7x - 8y)(7x - 8y)$.
29. $(7ab - 2c)(7ab - 2c)$.
30. $(11x - 13y)(11x - 13y)$.
31. $(2a^2 + 5x^2)(2a^2 + 5x^2)$.
32. $(x + y - 2z)(x + y - 2z)$.
33. $(a - b - 3)(a - b - 3)$.
34. $(a + c + 5)(a + c + 5)$.

Exercise 39. Page 90.

1. $(a + 2)(a - 2)$.
2. $(1 + x)(1 - x)$.
3. $(x + 3y)(x - 3y)$.
4. $(2a + 7b)(2a - 7b)$.

5. $(x+2y)(x-2y)$.
6. $(7+10y)(7-10y)$.
7. $(1+7x^3)(1-7x^3)$.
8. $(2+11y^4)(2-11y^4)$.
9. $(1+13a^3)(1-13a^3)$.
10. $(ab+2c^3)(ab-2c^3)$.
11. $(3x^4+a^3)(3x^4-a^3)$.
12. $(2x^8+y^{10})(2x^8-y^{10})$.
13. $(5+4a)(5-4a)$.
14. $(4+5y)(4-5y)$.
15. $(ab+1)(ab-1)$.
16. $(x+10)(x-10)$.
17. $(11a+6b)(11a-6b)$.
18. $(7a^7+y^6)(7a^7-y^6)$.
19. $(8a+3b^3)(8a-3b^3)$.
20. $(9a^2b^2+c^2)(3ab+c)(3ab-c)$.
21. $c(2a+3c^2)(2a-3c^2)$.
22. $5ab(2ab+1)(2ab-1)$.
23. $3a^3(a+2c)(a-2c)$.
24. $9(a+3b)(a-3b)$.
25. $(9x+2y)(9x-2y)$.
26. $(8a^2+b^2)(8a^2-b^2)$.
27. $(mn+6)(mn-6)$.
28. $(x^2+12)(x^2-12)$.
29. $(x+5)(x-5)$.
30. $(5+8y)(5-8y)$.
31. $x(4x^3+3y^3)(4x^3-3y^3)$.
32. $x^3(5x+4a^4)(5x-4a^4)$.
33. $a^2(6x+7a)(6x-7a)$.
34. $(x+4y)(x-4y)$.
35. $(1+20x^2)(1-20x^2)$.
36. $c(2a+3c)(2a-3c)$.

Exercise 40. Page 91.

1. $(x+y+z)(x+y-z)$.
2. $(x-y+z)(x-y-z)$.
3. $(x-2y+2z)(x-2y-2z)$.
4. $(a+3b+4c)(a+3b-4c)$.
5. $(a-b+c-d)(a-b-c+d)$.
6. $(2a+b+5c)(2a+b-5c)$.
7. $(3x+y)(3y-x)$.
8. $(4x-1)(7-2x)$.
9. $(x+y-z)(x-y+z)$.
10. $(a+3b-2c)(a-3b+2c)$.
11. $(b+2a+3c)(b-2a-3c)$.
12. $(1+x+5b)(1-x-5b)$.
13. $(3a+x-3c)(3a-x+3c)$.
14. $(4a+2y-3z)(4a-2y+3z)$.
15. $4b(a-c)$.
16. $(5x-4a)(x+2a)$.
17. $-5a(a+2)$.
18. $-y(2x+y-10)$.
19. $(3a-b)(a+3b-2c)$.
20. $4(a+b+c)(b-4c)$.

Exercise 41. Page 93.

1. $(a+b+2c)(a+b-2c)$.
2. $(x-y+3a)(x-y-3a)$.
3. $(b+x-2a)(b-x+2a)$.
4. $(2a+b+x)(2a+b-x)$.
5. $(a+x+y)(a-x-y)$.
11. $(3x-1+a+2b)(3x-1-a-2b)$.
12. $(a+b+x+3y)(a+b-x-3y)$.
13. $(x-1+b-y)(x-1-b+y)$.
14. $(3-x+a+4b)(3-x-a-4b)$.
17. $(2a-3c+3b-2d)(2a-3c-3b+2d)$.
18. $(2x-1+2y-z)(2x-1-2y+z)$.
6. $(1+a+b)(1-a-b)$.
7. $(a+b+4ab)(a+b-4ab)$.
8. $(2x+3a-1)(2x-3a+1)$.
9. $(a-b+c+d)(a-b-c-d)$.
10. $(x-y+a+b)(x-y-a-b)$.
15. $(2-x+b+2a)(2-x-b-2a)$.
16. $(a^2-b^2+a-3)(a^2-b^2-a+3)$.

19. $(1+x-2y)(1-x+2y)$. 20. $(a-c+b-1)(a-c-b+1)$.
 21. $(a+2c+3x+1)(a+2c-3x-1)$.
 22. $(2+3x-2y)(2-3x+2y)$.
 23. $(2x^2-3+2y-3z)(2x^2-3-2y+3z)$.
 24. $(a^2-3c+b-2x^2)(a^2-3c-b+2x^2)$.
 25. $(5a-b+3ax-1)(5a-b-3ax+1)$.
 26. $(4x^2+c^2+5x^3-3)(4x^2+c^2-5x^3+3)$.
 27. $(2a^2-3b^3+9xy^2z^3)(2a^2-3b^3-9xy^2z^3)$.

Exercise 42. Page 94.

1. $(x^2+xy+y^2)(x^2-xy+y^2)$. 3. $(3a^2+3a-1)(3a^2-3a-1)$.
 2. $(x^2+x+1)(x^2-x+1)$. 4. $(a+1)(a-1)(4a+1)(4a-1)$.
 5. $(2a^2+3a-1)(2a^2-3a-1)$.
 6. $(3a^2+2ab+5b^2)(3a^2-2ab+5b^2)$.
 7. $(2x^2+3xy-3y^2)(2x^2-3xy-3y^2)$.
 8. $(2a^2+3ac-5c^2)(2a^2-3ac-5c^2)$.
 9. $(2a^2+2ac+5c^2)(2a^2-2ac+5c^2)$.
 10. $(5x^2+3xy+4y^2)(5x^2-3xy+4y^2)$.

Exercise 43. Page 97.

1. $(x+3)(x+5)$. 15. $(x+1)(x-5)$. 29. $(x+y)(x+4y)$.
 2. $(x-3)(x-5)$. 16. $(x+3)(x+6)$. 30. $(x+y)(x-4y)$.
 3. $(x-3)(x+5)$. 17. $(x-3)(x-6)$. 31. $(x-y)(x-4y)$.
 4. $(x-5)(x+2)$. 18. $(x-3)(x+6)$. 32. $(x-y)(x+4y)$.
 5. $(x+2a)(x+3a)$. 19. $(x+3)(x-6)$. 33. $(x+y)(x+2y)$.
 6. $(x-2a)(x-3a)$. 20. $(x+1)(x+8)$. 34. $(a-2b)(a-5b)$.
 7. $(x+3)(x-5)$. 21. $(x-1)(x-8)$. 35. $(ax+6)(ax-9)$.
 8. $(x+2)(x+3)$. 22. $(x-1)(x+8)$. 36. $(x+4)(x-11)$.
 9. $(x-2)(x-3)$. 23. $(x+1)(x-8)$. 37. $(x-11)(x+12)$.
 10. $(x-2)(x+3)$. 24. $(x+2)(x+5)$. 38. $(x-5)(x-10)$.
 11. $(x+2)(x-3)$. 25. $(x-2)(x-5)$. 39. $(a-8)(a-15)$.
 12. $(x+1)(x+5)$. 26. $(x-2)(x+5)$. 40. $(a-13)(a+30)$.
 13. $(x-1)(x-5)$. 27. $(x-2a)(x+3a)$. 41. $(c-5)(c+30)$.
 14. $(x-1)(x+5)$. 28. $(x+2a)(x-3a)$. 42. $(c-1)(c-57)$.
 43. $(a^2-5b^3)(a^2-6b^3)$. 45. $(xy+3z)(xy+16z)$.
 44. $(z+4y)(z+5y)$. 46. $(ab-2c)(ab-11c)$.

Exercise 44. Page 99.

1. $(x+1)(2x+3)$. 4. $(2x+1)(3x+2)$. 7. $(2x-1)(4x-3)$.
 2. $(3x+2)(x-1)$. 5. $(2x+1)(3x-2)$. 8. $(6x-1)(3x-2)$.
 3. $(5x-3)(x-1)$. 6. $(5x-2)(3x+4)$. 9. $(4x+1)(3x-2)$.

10. $(4x-1)(3x-1)$. 13. $(3x-2)(x+2)$. 16. $(4x-1)(x+3)$.
 11. $(4x+1)(3x-1)$. 14. $(2x-1)(3x+4)$. 17. $(2x+1)(2x-3)$.
 12. $(x+1)(3x-5)$. 15. $(4x+1)(x+3)$. 18. $(2x+1)(2x+3)$.
 19. $(3ax-1)(2ax+1)$. 20. $(3x+4)(2x+3)$.
 21. $(3x+2)(4x-7)$. 23. $(8c-3)(c+7)$. 25. $(2x+y)(x+2y)$.
 22. $(5a+1)(2a-5)$. 24. $(8z+3)(z-5)$. 26. $(3x+b)(2x-3b)$.
 27. $(4x-3b)(2x+5b)$. 38. $(5x+2y)(2x-5y)$.
 28. $(3a-2c)(2a-5c)$. 39. $(7x-3y)(2x-7y)$.
 29. $(4x+3y)(2x+7y)$. 40. $(3x-4y)(2x-5y)$.
 30. $(4x+3y)(2x-7y)$. 41. $(6x-y)(x+6y)$.
 31. $(3x-y)(2x+7y)$. 42. $(4x+y)(6x-5y)$.
 32. $(11a-b)(a-2b)$. 43. $(4x-3y)(6x-5y)$.
 33. $(2c-d)(c-6d)$. 44. $(4x+3y)(6x-5y)$.
 34. $(3y-z)(2y+3z)$. 45. $(9x+2y)(4x-3y)$.
 35. $(5x-2y)(3x-4y)$. 46. $(5x+3y)(3x+2y)$.
 36. $(3x-2y)(3x+4y)$. 47. $(12x-5y)(x+3y)$.
 37. $(3x+7y)(2x-5y)$. 48. $(5y-2)(y+3)$.

Exercise 45. Page 101.

1. $(a+2b)(a^2-2ab+4b^2)$. 4. $(5a+1)(25a^2-5a+1)$.
 2. $a^3(1-3a)(1+3a+9a^2)$. 5. $(3xy-1)(9x^2y^2+3xy+1)$.
 3. $(a+4)(a^2-4a+16)$. 6. $(a+3b)(a^2-3ab+9b^2)$.
 7. $(xy-4)(x^2y^2+4xy+16)$.
 8. $(4a^2+5b)(16a^4-20a^2b+25b^2)$.
 9. $(6a^2-b)(36a^4+6a^2b+b^2)$. 12. $(ab+7)(a^2b^2-7ab+49)$.
 10. $(4a-3b)(16a^2+12ab+9b^2)$. 13. $(2a-b^2)(4a^2+2ab^2+b^4)$.
 11. $(7-x)(49+7x+x^2)$. 14. $(6m+n^2)(36m^2-6mn^2+n^4)$.
 15. $(xy-8z)(x^2y^2+8xyz+64z^2)$.
 16. $27(3a^2+2c^2)(9a^4-6a^2c^2+4c^4)$.
 17. $(9y-4z)(81y^2+36yz+16z^2)$.
 18. $(8a-1)(64a^2+8a+1)$.
 19. $(a+b-1)(a^2+2ab+b^2+a+b+1)$.
 20. $(a-b+1)(a^2-2ab+b^2-a+b+1)$.
 21. $(1-a+b)(1+a-b+a^2-2ab+b^2)$.
 22. $(3x^2+5y)(9x^4-15x^2y+25y^2)$.
 23. $(6+7y^2)(36-42y^2+49y^4)$. 24. $x^6(x^3-5)(x^6-5x^3+25)$.
 25. $(2x^6+3y^4)(4x^{12}-6x^6y^4+9y^8)$.
 26. $(a^5-6b^3)(a^{10}+6a^5b^3+36b^6)$.
 27. $(cd-7z^2)(c^2d^2+7cdz^2+49z^4)$.
 28. $(xyz^2+2)(x^2y^2z^4-2xyz^2+4)$.

29. $(2xy^2z^3 - 1)(4x^2y^4z^6 + 2xy^2z^3 + 1)$.
 30. $(3x^3yz^2 + 1)(9x^6y^2z^4 - 3x^3yz^2 + 1)$.
 31. $(4x - 5y^3)(16x^2 + 20xy^3 + 25y^6)$.
 32. $(5x^2 + 3y^6)(25x^4 - 15x^2y^6 + 9y^{10})$.
 33. $(x + y)(7x^2 - 4xy + y^2)$.
 34. $(2x + 2y + z)(4x^2 + 8xy + 4y^2 - 2xz - 2yz + z^2)$.
 35. $2b(3a^2 + b^2)$.
 36. $(x^2 - 3 - z)(x^4 - 6x^2 + 9 + x^2z - 3z + z^2)$.
 37. $(b + a - c)(b^2 - ab + bc + a^2 - 2ac + c^2)$.
 38. $(x + z)(x^2 - 3x + xz - 3z + z^2 + 3)$.
 39. $(a - 3b - c)(a^2 - 6ab + 9b^2 + ac - 3bc + c^2)$.
 40. $(2 - 3a + b)(4 - 12a + 9a^2 - 2b + 3ab + b^2)$.
 41. $(z + 5 - a)(z^2 + 10z + 25 + az + 5a + a^2)$.
 42. $2y(y^2 + 3z^2)$.
 43. $(x + 2y)(7x^2 + xy + y^2)$.
 44. $(1 - 7a + 5b)(1 + 7a - 5b + 49a^2 - 70ab + 25b^2)$.
 45. $(3x + y - 2z)(9x^2 + 6xy + y^2 + 6xz + 2yz + 4z^2)$.
 46. $(2a - b - c)(4a^2 - 4ab + b^2 - 2ac - bc + c^2)$.
 47. $(x - y + z)(x^2 + xy - xz + y^2 - 2yz + z^2)$.
 48. $(5 - x + 2y)(25 + 5x - 10y + x^2 - 4xy + 4y^2)$.
 49. $(2x + y - 3z)(4x^2 + 4xy + y^2 + 6xz + 3yz + 9z^2)$.
 50. $(a - 3 - z)(a^2 - 6a + 9 + az - 3z + z^2)$.
 51. $(7a - b + c)(49a^2 - 14ab + b^2 - 7ac + bc + c^2)$.
 52. $(3c - 2y - 2z)(9c^2 - 12cy + 4y^2 + 6cz - 4yz + 4z^2)$.

Exercise 46. Page 103.

- | | |
|--------------------------------|---------------------------------|
| 1. $(x + 3)(x^2 - 3x - 1)$. | 10. $(x - 1)(x^2 - 2x + 2)$. |
| 2. $(x - 5)(x^2 + 5x - 1)$. | 11. $(x + 2)(x^2 + 7x + 2)$. |
| 3. $(x - 4)(x^2 + 4x + 1)$. | 12. $(x - 5)(x^2 + 7x + 1)$. |
| 4. $(x + 3)(x^2 - 3x + 1)$. | 13. $(x - 1)(4x^2 - 8x + 1)$. |
| 5. $(x + 3)(x^2 - x + 3)$. | 14. $(x - 3)(x - 4)(x + 5)$. |
| 6. $(x - 1)(x - 1)(x + 2)$. | 15. $(x - 3)(2x - 1)(3x - 1)$. |
| 7. $(x - 2)(x - 2)(x + 4)$. | 16. $(x - 3)(x - 3)(x - 4)$. |
| 8. $(x - 1)(x^2 + 5x + 5)$. | 17. $(x + 2)(x^2 + 5x + 2)$. |
| 9. $(x - 1)(2x - 1)(2x + 3)$. | 18. $(x + 2)(x^2 + 3x + 1)$. |

Exercise 47. Page 105.

- | | |
|---|--|
| 1. $a(a + 3)(a - 3)$. | 3. $(x^4 + y^4)(x^8 - x^4y^4 + y^8)$. |
| 2. $x^2(2x + 1)(2x - 1)$. | 4. $(x^2 + y^2)(x + y)(x - y)$. |
| 5. $(x + y)(x^2 - xy + y^2)$. | |
| 6. $(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$. | |

7. $(6x + 7y)(6x - 7y)$.
8. $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$.
9. $(x - 7)(x - 7)$.
10. $(x + a - b)(x - a + b)$.
11. $(a + m + n)(a - m - n)$.
12. $(x - 2)(x - 9)$.
13. $(x - 5)(x + 9)$.
14. $(x + 4)(x + 9)$.
15. $(x + 3)(x - 16)$.
16. $(x - 3)(x - 12)$.
17. $(x - 10)(x + 11)$.
18. $(2x - y)(x + 2y)$.
19. $(z + 4)(z - 10)$.
20. $(x^2 + xy + y^2)(x^2 - xy + y^2)$.
21. $(x + 5)(x - 12)$.
22. $(2a - 3)(a - 2)$.
23. $xy^2(x - 4y^2 - 3xy)$.
24. $(x^2 + 1)(x + 1)$.
25. $(3x - 2)(x + 1)$.
26. $(x - 5)(x - 19)$.
27. $(3a + 2)(3a + 2)$.
28. $(a + b - c)(a - b + c)$.
29. $(x + 2)(x - y)$.
30. $(x^2 - 3)(3x + 2)$.
31. $(x - y + b)(x - y - b)$.
32. $(m - n + 1)(m - n - 1)$.
33. $(x + 3b)(x + 2a)$.
34. $(x - m + n)(x - m - n)$.
35. $(3x^2 + 3xy + 5y^2)(3x^2 - 3xy + 5y^2)$.
36. $(x + y + 2)(x + y - 2)$.
37. $(2x + 7)(3x - 11)$.
38. $(x + 5y)(x - 13y)$.
39. $(x^2 + 3x + 1)(x^2 - 3x + 1)$.
40. $(1 + a + b)(1 - a - b)$.
41. $3x^2(x^2 - 2x + 3)$.
42. $(x^2 - 2)(x - 5)$.
43. $(x - b)(x + a)$.
44. $(x + 2a)(2x - 3y)$.
45. $(x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$.
46. $(2x - c)(16x^4 + 8cx^3 + 4c^2x^2 + 2c^3x + c^4)$.
47. $(a^2 + 4y^2)(a^4 - 4a^2y^2 + 16y^4)$.
48. $(3 + x)(9 - 3x + x^2)(3 - x)(9 + 3x + x^2)$.
49. $(x^2 + y^2)(x^4 - x^2y^2 + y^4)(x + y)(x - y)(x^2 + xy + y^2)(x^2 - xy + y^2)$.
50. $(a^2 + 2ab + b^2 + 1)(a + b + 1)(a + b - 1)$.
51. $(4x^2 + 9)(2x + 3)(2x - 3)$.
52. $(x^2 + x + 1)(x^2 - x + 1)$.
53. $(3x - 4a)(9x^2 + 12ax + 16a^2)$.
54. $(x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$.
55. $(x - y)(x^2 + xy + y^2)(x^6 + x^3y^3 + y^6)$.
56. $(a^4 + 16)(a^2 + 4)(a + 2)(a - 2)$.
57. $(1 - x + y)(1 + x - y + x^2 - 2xy + y^2)$.
58. $(x^2 - 6)(x^4 + 6x^2 + 36)$.
59. $(a + 7)(a - 11)$.
60. $(a + b)(a^2 - ab + b^2 + 1)$.
61. $(a - b)(a^2 + ab + b^2 + 1)$.
62. $(a - b)(a + b + 1)$.
63. $(x - 1)(x^2 + x + 1)(ax + b)$.
64. $(x - y)(x - 3y)$.
65. $(1 - 5xy)(1 - 5xy)$.
66. $(a + b - c)(a - b + c)$.
67. $(x - 2y + z)(x - 2y - z)$.
68. $(a + 2b - 3c)(a - 2b + 3c)$.
69. $(2x - 3y + z)(2x - 3y - z)$.
70. $(a + b + c - d)(a + b - c + d)$.
71. $(a + 3b)(a - 3b + 1)$.
72. $5c^2(c + 3)(c - 6)$.
73. $(x + y)(a + c)(a - c)$.
74. $(x^2 + 4ax + 16a^2)(x^2 - 4ax + 16a^2)$.

75. $9x(x^2 - xy + y^2)$.
 76. $(a + b + c)(a + b - c)$.
 77. $(y + a + c)(y - a - c)$.
 81. $(a - b + 2x - 3y)(a - b - 2x + 3y)$.
 83. $(a + b + 1)(a + b - ab - 1)$.
 84. $(x + 1)(x^2 - 2x + 5)$.
 87. $(a + b + c)(a + b - c)(c + a - b)(c - a + b)$.
 88. $(x - 1)(x + 3)$.
 89. $(x - 5)(x + 8)$.
 90. $(x + 1)(x - 10)$.
 94. $(x - 2a)(x - 8a)$.
 96. $(1 + x)(1 - 10x)$.
 97. $(x + 3y)(x + 4y)$.
 100. $(x^2 + y^2 + 2yz + z^2)(x + y + z)(x - y - z)$.
 101. $(x - y)(x - y)(x - y)$.
 102. $(x + 1)(x - 1)(x - 2)$.
 103. $(x - 2)(x + 1)(x + 4)$.
 104. $3y^2(x - y)(x + 4y)$.
 105. $(a + b)(a - b)(a^2 - ab + b^2)$.
 106. $(a + 2c + 1)(a - 2c)$.
 107. $(2a + 3b + c)(2a + 3b - c)$.
 108. $(5x + b)(3x - a)$.
 117. $(a + b - c)(a - b + c + 1)$.
 118. $(x - 2y)(x^2 + 2xy + 4y^2)(z + 2n)(z - 2n)$.
 119. $(b + c)(5a + 3b + 1)$.
 121. $(x^2 - ab + a^2 + b^2)(x^2 - ab - a^2 - b^2)$.
 78. $x(x + 10a)$.
 79. $(z + x - y)(z - x + y)$.
 80. $(2a^2 + 3a - 1)(2a - 1)(a - 1)$.
 82. $2(x - y)(x - y + a)$.
 85. $(x + y + z)(x - y - z + 1)$.
 86. $(x - y - z)(x - y - z)$.
 91. $(x + 1)(x - 1)(x^2 + 9)$.
 92. $x^2(x + 4)(x - 6)$.
 93. $(x^2 + 3)(x^2 - 17)$.
 95. $(x^2 + 3xy - 3y^2)(x^2 - 3xy - 3y^2)$.
 98. $(x - y)(x + 2y)$.
 99. $(x - y)(x + 4y)$.
 109. $3a^2(a^2 + 5ab - 8b^2)$.
 110. $6a(a - 2b)(a - 3b)$.
 111. $(5a + 2x - 2)(5a - 2x)$.
 112. $xy(x + y)(x - y)(x - y)$.
 113. $(3a - 2b + 1)(3a + 2b)$.
 114. $(x - y)(x^2 + xy + y^2 - 2x)$.
 115. $(x - y - 1)(x - y - 1)$.
 116. $(a + 2)(a - 2)(a - 2c + 1)$.

Exercise 48. Page 110.

- | | | |
|---------------------|------------------------|--------------------|
| 1. $24a^2$. | 11. $x + 4$. | 21. $x + 3$. |
| 2. $9x^3$. | 12. $x - 4$. | 22. $x - 5$. |
| 3. $6a^2x^2$. | 13. $x - 4$. | 23. $3a^2 - 9ab$. |
| 4. $4x^2$. | 14. $3x - 2y$. | 24. $3xy - 3y^2$. |
| 5. $12a^2c$. | 15. $x + 1$. | 25. $1 + 4c^2$. |
| 6. $2(a + b)^2$. | 16. $x + 5y$. | 26. $3x - 1$. |
| 7. $4(x + y)^2$. | 17. $x^2(x - 6)$. | 27. $x + 6$. |
| 8. $(x + 2)^2$. | 18. $x - 3y$. | 28. $2x + 3y$. |
| 9. $6a^2b(a + b)$. | 19. $1 - 4x + 16x^2$. | 29. $x + y$. |
| 10. $x^2 - 3x$. | 20. $x^2 + 9$. | |

Exercise 49. Page 112.

- | | | |
|-------------------|-----------------|-----------------|
| 1. x^3y^3 . | 3. $12x^3y^3$. | 5. $84x^3y^7$. |
| 2. $5a^2b^2c^3$. | 4. $10a^3b^5$. | 6. $30x^2z^3$. |

7. $a^2b^3c^3$. 9. $x^2(x+1)$. 11. $ab(a+b)$.
 8. $x^2y^3z^2$. 10. $x(x+1)(x-1)$. 12. $a(a+2)^2$.
 13. $(x+2)^2(x+3)$. 20. $(x+2)(x+3)(x+4)$.
 14. $(x-4)(x+5)(x-6)$. 21. $(1+y)(1-y)(1+y+y^2)$.
 15. $(y-5)(y-6)(y+7)$. 22. $(x+y)^2(x-y)^2$.
 16. $(z-4)(z+5)(z-6)$. 23. $x(x-3)(x+5)(x^2+3x+9)$.
 17. $(a+b)^2(a-b)^2$. 24. $(y+1)(y-1)(y^2+1)$.
 18. $(a+2c)^2(a-2c)^2$. 25. $(x+y+z)^2(x+y-z)$.
 19. $4x^3y(x+y)^2(x-y)$. 26. $(x-a)(x-b)(x-c)$.
 27. $(x+y)(x+2y)(x+4y)(x-7y)$.
 28. $(x-2y)(x-3y)(x-4y)$.

Exercise 50. Page 119.

1. H. C. F. = $4x-5$;
 L. C. M. = $(4x-5)(x+2)(x^2+3x+3)$.
 2. H. C. F. = $x-2$;
 L. C. M. = $(x-2)(2x^2-2x+1)(8x^2-7x+3)$.
 3. H. C. F. = $3x+4a$;
 L. C. M. = $x(3x+4a)(x-a)(2x-5a)$.
 4. H. C. F. = $3x-7$;
 L. C. M. = $(3x-7)(x^2-2x+3)(2x-1)(x+3)$.
 5. H. C. F. = c^2-c-1 ;
 L. C. M. = $2c(c^2-c-1)(c-1)(c^2+1)$.
 6. H. C. F. = $a^2-4ax+4x^2$;
 L. C. M. = $2(a-2x)^3$.
 7. H. C. F. = $7x^2+5x+5$;
 L. C. M. = $(7x^2+5x+5)(x-1)(x+1)$.
 8. H. C. F. = x^2-x-6 ;
 L. C. M. = $(x+1)(x-1)(x+2)(x-2)(x+3)(x-3)$.
 9. H. C. F. = $2x^2-x-5$;
 L. C. M. = $(2x^2-x-5)(x+2)(2x-1)$.
 10. H. C. F. = $3x+2$;
 L. C. M. = $(3x+2)(2x^2-2x-3)(4x^2-3x-8)$.
 11. H. C. F. = $2x+1$;
 L. C. M. = $(2x+1)(x+1)(x-1)(3x+2)(3x-2)$.
 12. H. C. F. = $x^2-7x+12$;
 L. C. M. = $(x-3)(x-4)(x-2)(x-5)$.

13. H. C. F. = $2x - 7$;
L. C. M. = $(2x - 7)(2x^2 + 6x + 13)(6x^2 + 7x + 6)$.
14. H. C. F. = $x^2 - 2x + 3$;
L. C. M. = $(x^2 - 2x + 3)^2(x^2 + 2x + 3)$.
15. H. C. F. = $x^2 - 8$;
L. C. M. = $2x(x^2 - 8)(2x - 3)(2x^2 + x + 2)$.
16. H. C. F. = $6x - 1$;
L. C. M. = $(6x - 1)(2x^2 + x + 3)(4x^2 - 8x + 1)$.
17. H. C. F. = $x^2 + 2ax - a^2$;
L. C. M. = $(x^2 + 2ax - a^2)(2x + 3a)(4x + a)$.
18. H. C. F. = $x^2 - ax + a^2$;
L. C. M. = $4(x^2 - ax + a^2)(2x - 7a)(x - 4a)$.
19. H. C. F. = $x^2 + 5x + 1$;
L. C. M. = $(x^2 + 5x + 1)(2x^2 - x + 3)(3x^2 - x + 2)$.
20. H. C. F. = $2x^2 - 3x + 3$;
L. C. M. = $2(2x^2 - 3x + 3)(5x + 8)(10x^2 + 15x - 1)$.
21. H. C. F. = $2x - 3$;
L. C. M. = $(2x - 3)(x + 3)(x - 1)(x^2 - 2x + 3)$.
22. H. C. F. = $x^2 - 3x + 1$;
L. C. M. = $(x + 3)(x^2 - 3x + 1)(x^4 - x^2 - 3x - 8)$.
23. H. C. F. = $x - y$;
L. C. M. = $x(x - y)(4x + 3y)(3x^2 + y^2)$.
24. H. C. F. = $4x - 5$;
L. C. M. = $(4x - 5)(2x^3 + x^2 + x + 5)(x^2 + 3x + 3)$.
25. H. C. F. = $2x - 3$;
L. C. M. = $(2x - 3)(x + 1)(2x - 1)(5x - 2)$.
26. H. C. F. = $3x - 2$;
L. C. M. = $2x(3x - 2)(2x^2 - 3x - 1)(x^3 - x^2 - 1)$.
27. H. C. F. = $2x^2 - x + 3$;
L. C. M. = $(2x^2 - x + 3)(x^2 - x - 1)(x + 1)(2x - 1)$.
28. H. C. F. = $3x^2 - x + 4$;
L. C. M. = $(3x^2 - x + 4)(x^2 - 2)(2x + 5)$.
29. H. C. F. = $3x^2 - x + 1$;
L. C. M. = $x^2(3x^2 - x + 1)(x + 1)(x^2 + x - 1)$.
30. H. C. F. = $x + 3$;
L. C. M. = $(x + 3)(2x^2 - x + 1)(x - 1)(3x - 4)$.

31. H. C. F. $= x(2x^2 - 3x + 5)$;
L. C. M. $= 2x(2x^2 - 3x + 5)(2 - 3x^2)(x^2 + 4x + 6)$.
32. H. C. F. $= x^2 - 4x - 1$;
L. C. M. $= (x^2 - 4x - 1)(x^2 + 3x - 1)(x^3 - 2)$.
33. H. C. F. $= a(a - 2)$;
L. C. M. $= a(a - 2)(2a^2 + 2a + 1)(3a^2 + 5a + 8)$.
34. H. C. F. $= x - 2a$;
L. C. M. $= 2(x - 2a)(3x^2 - ax + a^2)(x^2 + a^2)(x + a)$.
35. H. C. F. $= 3x^2 + x - 2$;
L. C. M. $= (x + 1)(3x - 2)(x^2 - 3x^3 - 2)(2x^2 - 3x - 1)$.
36. H. C. F. $= a(2a - 3x)$;
L. C. M. $= a^2x(2a - 3x)(a + 3x)(a - x)(3a - x)$.
37. H. C. F. $= a(a^2 - 2a + 1)$;
L. C. M. $= 6a^2(a - 1)^2(a^2 + 3)(a^2 + a - 1)$.

Exercise 51. Page 122.

1. H. C. F. $= 2x - 1$;
L. C. M. $= (2x - 1)(3x + 2)(x + 4)(x - 3)$.
2. H. C. F. $= a + b$;
L. C. M. $= (a + b)^2(a - b)(a^2 + ab + b^2)$.
3. H. C. F. $= x - a$;
L. C. M. $= (x - a)(x - 4a)(x - 2a)(3x - 7a)$.
4. H. C. F. $= x - 2$;
L. C. M. $= (x - 2)(x + 3)(x - 1)(x + 1)(x + 4)$.
5. H. C. F. $= x - 3$;
L. C. M. $= (x - 3)(x - 1)(x - 2)(x - 4)$.
6. H. C. F. $= 1$;
L. C. M. $= (2x + 3y)(3x - y)(x + 4y)$.
7. H. C. F. $= 1$;
L. C. M. $= 2a^2(1 - a)(4 - 3a)(2 - a)(2 + 3a)$.
8. H. C. F. $= 1$;
L. C. M. $= 3x(2x + 3)(3x - 1)(x + 5)$.
9. H. C. F. $= 3x - a$;
L. C. M. $= (3x - a)(9x^2 + 3ax + a^2)(2x + a)(5x + b)$.
10. H. C. F. $= x - 1$;
L. C. M. $= (x - 1)(x^2 + x + 1)(2x + 1)(3x + 2)$.
11. H. C. F. $= 1$;
L. C. M. $= (3x - 2)(2x + 1)(7x - 1)$.

12. H. C. F. = 2 ;

L. C. M. = $12(2x-1)(3x+2)(2x+1)(x-4)(x-3)$.

13. H. C. F. = $x-1$;

L. C. M. = $(x-1)(2x+5)(3x+2)(2x+3)$.

14. H. C. F. = $x+1$;

L. C. M. = $(x+1)^2(x-1)(3x-1)$.

Exercise 52. Page 124.

1. $\frac{2b}{3a}$

5. $\frac{5yz^2}{3x}$

9. $\frac{2c}{3a^2b}$

13. $\frac{b}{b+1}$

2. $\frac{1}{5ac}$

6. $\frac{3n^2}{4p}$

10. $\frac{3y}{4x}$

14. $5(a-c)$

3. $\frac{2x}{3y^2}$

7. $\frac{2x^2}{3ay^5}$

11. $\frac{2a-3c}{2a}$

15. $\frac{x^2-xy+y^2}{x+y}$

4. $\frac{6mb}{7n^2}$

8. $7a^2bc$

12. $\frac{3a}{a+2}$

16. $\frac{x^2+3x+9}{x+5}$

17. $\frac{x-5}{2x-7}$

18. $\frac{x+4}{2x+3}$

28. $\frac{x^2-2x-2}{3x^2-6x+4}$

19. $\frac{2x+3a}{4x^2-6ax+9a^2}$

29. $\frac{x^2+x-2}{x^2+5x+5}$

20. $\frac{x-y-z}{x+y-z}$

23. $\frac{a+b-c}{a+b+c}$

30. $\frac{3x^2+2x+1}{3x^2-x-2}$

21. $\frac{x^2-xy+y^2}{x-y}$

24. $\frac{x-a+b}{x+a-b}$

31. $\frac{x^2+x-6}{x^2-1}$

22. $\frac{2a+3}{3a+5}$

25. $\frac{x+a-b}{x-a+b}$

32. $\frac{x^2+5x+4}{2x^2+7x+3}$

26. $\frac{a+b-c-d}{a+c-b-d}$

33. $\frac{x-5}{x+5}$

34. $\frac{2x-3}{3x+2}$

27. $\frac{1}{(a+b-c)(b-a+c)}$

35. $\frac{x+6}{x^2+x-2}$

Exercise 53. Page 127.

1. $-\frac{y}{x^2+3xy+9y^2}$

5. $-\frac{x+7}{x+2}$

8. $-\frac{a-b}{x+c}$

2. $-\frac{b}{c}$

6. $-\frac{3(a+b)}{4}$

9. $-\frac{a+b+c+d}{a-b+c+d}$

3. $-\frac{2a}{2a+3b}$

7. $-\frac{a+b+c}{a}$

10. $-\frac{x-a-b}{x+a-b}$

4. $-\frac{x+1}{4x}$

Exercise 54. Page 128.

1. $x + 3 - \frac{3}{4x}$.

6. $4x + 13 - \frac{2}{x-4}$.

2. $x - 2 - \frac{2}{3x}$.

7. $3x - 4 - \frac{3x+14}{x^2+x-3}$.

3. $x^2 + xy + y^2 + \frac{2y^3}{x-y}$.

8. $x + 3 - \frac{x+1}{x^2+x-2}$.

4. $x^2 - xy + y^2 - \frac{2y^3}{x+y}$.

9. $2x + 6a - \frac{9a^2+2}{2x-3a}$.

5. $a^2 - 2ax + 4x^2 - \frac{6x^3}{a+2x}$.

10. $x + 1 - \frac{3x-6}{5x^2+4x-1}$.

Exercise 55. Page 129.

1. $\frac{a^2+b^2}{a+b}$.

3. $\frac{x^2}{x-3}$.

5. $-\frac{x^3}{a-x}$.

7. $\frac{ab}{a+b}$.

2. $\frac{2ax}{a+x}$.

4. $\frac{a^3}{a+x}$.

6. $\frac{5x-3a}{4}$.

8. $\frac{2x^2}{x-3}$.

Exercise 56. Page 131.

1. $\frac{3(3x-7)}{18}; \frac{4x-9}{18}$.

8. $\frac{1-2a}{1-4a^2}; \frac{1}{1-4a^2}$.

2. $\frac{3x(a-2x)}{9ax}; \frac{3x^2-2a}{9ax}$.

9. $\frac{9}{16-x^2}; \frac{(4-x)^2}{16-x^2}$.

3. $\frac{4(x-2y)}{10x^2}; \frac{x(3x-8y)}{10x^2}$.

10. $\frac{a^2}{27-a^3}; \frac{a(9+3a+a^2)}{27-a^3}$.

4. $\frac{12a(4a-5c)}{60a^2c}; \frac{5(3a-2c)}{60a^2c}$.

11. $\frac{4a^2+c^2}{4a^2-c^2}; \frac{(2a+c)^2}{4a^2-c^2}$.

5. $\frac{5(1+x)}{1-x^2}; \frac{6}{1-x^2}$.

12. $\frac{x^2+y^2}{25x^2-4y^2}; \frac{5x-2y}{25x^2-4y^2}$.

6. $\frac{x+3}{(x+2)(x+3)}; \frac{2(x+2)}{(x+2)(x+3)}$.

13. $\frac{(x+2)^2}{x^2-4}; \frac{(x-2)^2}{x^2-4}$.

7. $\frac{a(x+a)}{x^2-a^2}; \frac{a^2}{x^2-a^2}$.

14. $\frac{8x^2(a-b)}{6(a^2-b^2)}; \frac{xy}{6(a^2-b^2)}$.

15. $\frac{30(4x+1)}{15(x-2)}; \frac{5(2x-1)}{15(x-2)}; \frac{3(3x+2)}{15(x-2)}$.

16. $\frac{3(a-bm)}{12x}; \frac{12x}{12x}; \frac{4(c-bm)}{12x}$.

17. $\frac{a-c}{(a-b)(a-c)(b-c)}; \frac{b-c}{(a-b)(a-c)(b-c)}$.

18. $\frac{x(x+1)}{x^2-1}$; $\frac{x^2(x-1)}{x^3-1}$; $\frac{x^3}{x^2-1}$.
19. $\frac{a(a+b)}{a^2-b^2}$; $\frac{b(a-b)}{a^2-b^2}$; $\frac{c}{a^2-b^2}$.
20. $\frac{x+1}{(x+1)^2}$; $\frac{x}{(x+1)^2}$; $\frac{x^2}{(x+1)^2}$.
21. $\frac{x-c}{(x-c)(x-y)(x-z)}$; $\frac{x-z}{(x-c)(x-y)(x-z)}$; $\frac{x-y}{(x-c)(x-y)(x-z)}$.
22. $\frac{x-1}{(x-1)(x-2)(x-3)}$; $\frac{x-2}{(x-1)(x-2)(x-3)}$; $\frac{x-3}{(x-1)(x-2)(x-3)}$.
23. $\frac{x-3a}{x^2-5ax+6a^2}$; $\frac{1}{x^2-5ax+6a^2}$; $\frac{x-2a}{x^2-5ax+6a^2}$.

Exercise 57. Page 133.

1. $\frac{2x+2}{5}$. 6. $\frac{11-2x}{6}$. 10. $\frac{10x-18}{9x}$.
2. $\frac{x-4}{4}$. 7. $\frac{4b^2}{a^2}$. 11. $\frac{3x^2+5y^2+2xy}{x^3y^3}$.
3. $\frac{7x+5}{24}$. 8. $\frac{29x+16}{15}$. 12. $\frac{a^2b+b^2c+ac^2}{abc}$.
4. $\frac{36-25x}{36}$. 9. $\frac{14x-37}{40}$. 13. $\frac{1}{6y^2z}$.
5. $\frac{2x^2+24x-5}{8x^2}$.

Exercise 58. Page 134.

1. $\frac{2x+1}{x^2+x-30}$. 4. 0. 7. $\frac{9ac}{a^3-27c^3}$.
2. $\frac{2x}{x^2-1}$. 5. $\frac{2xy-2y^2}{(x+y)^2}$. 8. $\frac{2x^3}{1+x^2+x^4}$.
3. $\frac{1}{x-1}$. 6. $\frac{1}{a^2-b^2}$. 9. 0.
10. $\frac{2a^3x+2a^2x^2+2ax^3-2x^4}{a^4-x^4}$.
11. $\frac{2}{x^3+3x^2+2x}$. 14. $\frac{y}{x+y}$. 17. $\frac{2}{x}$.
12. $\frac{1}{a-3}$. 15. $\frac{x}{x^2-4y^2}$. 18. $\frac{3x^2-2ax-6a^2}{(x-a)^3}$.
13. $\frac{b^3}{(a+b)^3}$. 16. $\frac{2-y}{y(y^2-1)}$. 19. $\frac{3ax+7a^2}{x^3-8a^3}$.

20. $\frac{x^2 - 2x}{x^3 + 1}$ 22. $\frac{4}{4 - x^2}$ 24. $\frac{x - 4}{(x - 2)(x - 3)}$
 21. $\frac{3x^2 + 9}{x^3 - 27}$ 23. $\frac{5a}{x - 5a}$ 25. 0.
 26. $\frac{-17a}{(2a - 1)(5a + 3)(a + 4)}$ 27. $\frac{1}{(2 + 3x)(1 - 2x)(1 + x)}$

Exercise 59. Page 137.

1. $\frac{3x^2}{x^2 - 1}$ 3. $\frac{2a}{4a^2 - 9}$ 5. $\frac{2x - 3}{x(4x^2 - 1)}$
 2. $\frac{4a}{a + x}$ 4. $\frac{b}{a - b}$ 6. $\frac{3x^2 + a^2}{x^4 - 2a^2x^2 + a^4}$
 7. $\frac{2y^2}{x^3 - y^3}$ 8. 0. 9. -1. 10. 0.
 11. $\frac{10b - 9a - c}{(a - b)(b - c)(c - a)}$ 12. $\frac{1}{z(y - z)(z - x)}$ 13. 0.

Exercise 60. Page 139.

1. $\frac{8x}{3yz^3}$ 11. $\frac{1}{3(x - y)}$ 21. x^2 22. bcd .
 2. $\frac{5b^2mn^2}{7a^3c^2}$ 12. $\frac{(a - b)^2}{b}$ 23. $\frac{y(x - y)(x + y)^3}{x}$
 3. $\frac{10b^2mx^2}{7acy}$ 13. $\frac{x(a + 2x)}{a^2}$ 24. $\frac{1}{b}$
 4. $\frac{3mn}{10xy}$ 14. $\frac{x^2 + y^2}{x}$ 25. $\frac{1}{a}$ 26. $\frac{a}{b}$
 5. $\frac{4a^2b}{7xy}$ 15. $\frac{x^2 - ax + a^2}{x - 3a}$ 27. $\frac{a - b - c}{a + b + c}$
 6. $\frac{3y^2}{5z}$ 16. $\frac{a^2 + ab + b^2}{a + b}$ 28. $\frac{2x^2 + 2x + 1}{x}$
 7. $\frac{4x(x - y)}{x^2 + y^2}$ 17. $\frac{x + 5}{x + 3}$ 29. $\frac{1}{x - a}$
 8. $\frac{3x - 1}{x - 2}$ 18. $\frac{y(x^2 - y^2)}{x^3}$ 30. $\frac{a + b - c}{a}$
 9. $\frac{9}{20x}$ 19. $\frac{2x + y}{y}$ 31. $\frac{1}{a(a^2 - ab + b^2)}$
 10. $\frac{2a}{a - b}$ 20. $\frac{y - 2x}{y}$ 32. 1.

Exercise 61. Page 142.

- | | | |
|------------------------------------|------------------------------|---------------------------|
| 1. $\frac{20}{3yz}$ | 7. $\frac{x+a}{x-2a}$ | 13. $\frac{y}{x-y}$ |
| 2. $\frac{3ay}{8c}$ | 8. $\frac{xy}{2c+1}$ | 14. $\frac{3a}{2b(a-b)}$ |
| 3. $\frac{7a^3b^3c^3}{4m^4n^2p^7}$ | 9. $\frac{3x+2y}{2-x}$ | 15. $\frac{c-a+b}{c-a-b}$ |
| 4. $\frac{2acm}{3}$ | 10. $\frac{(x-2)(x-4)}{x^2}$ | 16. $\frac{x-a+b}{x+a-b}$ |
| 5. $\frac{6a^2b^2}{5c^4}$ | 11. 1. | 17. 1. |
| 6. abc . | 12. b . | 18. $\frac{x-y-z}{x+y+z}$ |

Exercise 62. Page 144.

- | | | | |
|---------------------------------------|-------------------------------|---|------------------------|
| 1. $\frac{x+y}{z}$ | 3. $-\frac{d}{c}$ | 5. $\frac{x+y}{x-y}$ | 7. $\frac{c^2+1}{c^3}$ |
| 2. $\frac{x^2z+xy}{xz^2-yz}$ | 4. $\frac{x+1}{x-1}$ | 6. $\frac{m+n}{m-n}$ | 8. 1. 9. 1 |
| 10. $\frac{ab^2}{a^3-a^2b-ab^2+b^3}$ | 11. $\frac{1}{c(a-b-c)}$ | 16. $\frac{x^2-y^2+1}{x^2-y^2+2}$ | |
| 12. $\frac{bc+ac+ab}{a^2c+ab^2+bc^2}$ | 14. $\frac{x^2-x+1}{2x-1}$ | 17. 1. | 18. 1. |
| 13. $-\frac{1}{bc(a-b+c)}$ | 15. $\frac{4}{3a+3}$ | 20. $-\frac{4xy(x^2+xy+y^2)}{(x^3+y^3)(x+y)}$ | |
| 19. $-\frac{m^2+mn+n^2}{mn}$ | 22. $\frac{x+x^2+x^4}{1+x^2}$ | 23. $\frac{x+y+z}{x-y+z}$ | |
| 21. $\frac{(b+c+a)^2}{2bc}$ | | | |

Exercise 63. Page 146.

- | | | | | |
|----------------------------------|--------------------|-------|--------------------------------------|-------------------------|
| 1. 8. | 2. $\frac{1}{2}$. | 3. 6. | 8. $\frac{x-y}{x(x+y)}$ | 10. $\frac{b}{a+b}$ |
| 4. $\frac{3x^2+4x-1}{3x^2+7x+3}$ | | | 9. $\frac{x-1}{x-2}$ | 11. abc . |
| 5. $\frac{x}{(x-1)(x-2)(x-3)}$ | | | 12. $\frac{(x+1)(x+2y)}{(x+2)(x+y)}$ | |
| 6. $\frac{8a^8}{a^8-b^8}$ | | | 13. 2. | 15. $\frac{2}{c(x+2c)}$ |
| 7. $\frac{12}{(a^4-4)(a^4-1)}$ | | | 14. $\frac{1}{2x^2-1}$ | 16. 2. |

17. $\frac{4x}{x^2-1}$ 20. $\frac{x+y}{x-y}$ 23. $\frac{b^2+a^2}{(b+a)^2}$
 18. 1. 21. 0. 24. $\frac{1}{1-x}$ 25. $x-2y$.
 19. 1. 22. 1. 26. $\frac{a^2+b^2+c^2}{abc}$.

Exercise 64. Page 149.

1. $x=-5$. 6. $x=7$. 11. $x=4$. 16. $x=-7$. 21. $x=7$.
 2. $x=-5$. 7. $x=7$. 12. $x=13$. 17. $x=1$. 22. $x=0$.
 3. $x=-\frac{3}{4}$. 8. $x=-\frac{1}{4}$. 13. $x=3$. 18. $x=\frac{1}{2}$. 23. $x=2$.
 4. $x=2$. 9. $x=\frac{1}{2}$. 14. $x=\frac{1}{2}$. 19. $x=8$. 24. $x=1$.
 5. $x=2$. 10. $y=24$. 15. $x=\frac{1}{17}$. 20. $x=4$. 25. $x=\frac{1}{2}$.

Exercise 65. Page 151.

1. $x=-9$. 3. $x=4$. 5. $x=3$. 7. $x=7$. 9. $x=4$.
 2. $x=8$. 4. $x=4$. 6. $x=-2$. 8. $x=4$. 10. $x=5$.

Exercise 66. Page 152.

1. $x = \frac{4a-2b}{a-3b}$. 8. $x = \frac{2n}{m}$. 15. $x = \frac{n}{n-1}$.
 2. $x = \frac{a^2-b^2}{2a}$. 9. $x = 1$. 16. $x = \frac{a^2(b-a)}{b^2+ab}$.
 3. $x = -\frac{ab}{a+b+c}$. 10. $x = \frac{a^2-b^2}{b-4a}$. 17. $x = \frac{ab}{a+b}$.
 4. $x = \frac{b(a+c)}{2b+c-a}$. 11. $x = \frac{abcd}{ab+bc+ac}$. 18. $x = \frac{2b^2-a^2}{4b-3a}$.
 5. $x = \frac{ab}{c}$. 12. $x = \frac{ab+c^2+a^2bc^2}{a^2-bc}$. 19. $x = \frac{bm(m+n)}{a}$.
 6. $x = \frac{bm-cn-dn}{b}$. 13. $x = \frac{3a^2-3ab}{8b-3a}$. 20. $x = \frac{a^2b+b^3}{ab^2-a^3}$.
 7. $x = \frac{2m}{n}$. 14. $x = 5a$. 21. $x = \frac{d}{c}$.
 22. $x = \frac{ab-ac+3b+3c}{b+c}$. 24. $x = \frac{ab}{b-a}$.
 23. $x = -\frac{2bc}{a^2+b^2}$. 25. $x = \frac{b^2c+b^2+c^2-ac}{b+c}$.
 26. $x = 0$. 27. $x = b$.

Exercise 67. Page 154.

1. 60. 3. 90. 6. 43; 20. 9. 66; 15. 12. 32; 7.
2. 45. 4. 42. 7. 85; 35. 10. 98; 11. 13. 11 years.
5. 51; 52. 8. 97; 29. 11. 407; 43. 14. 10 years.
15. A, 60 years; B, 12 years. 17. 20.
16. A, 50 years; B, 10 years. 18. 15.
19. Father, 64 years; son, 16 years.
20. 20. 21. A, 90 years; B, 30 years.
22. Father, 64 years; son, 11 years.
23. Length, 36 feet; breadth, 25 feet.
24. 576 square feet.
25. Length, 23 feet; breadth, 17 feet.
26. Length, 21 feet; breadth, 13 feet.
27. Length, 13 feet; breadth, 9 feet.
28. Length, 34 feet; breadth, 26 feet.
29. $1\frac{1}{2}$. 30. $1\frac{1}{4}$. 34. $10\frac{1}{11}$. 39. $\frac{10}{11}$ hour. 43. 63 miles.
31. $1\frac{3}{4}$. 35. 10. 36. 12. 40. $1\frac{1}{11}$. 44. 18.
32. $1\frac{1}{11}$. 37. $3\frac{2}{11}$. 41. 15 hours. 45. $6\frac{1}{2}$ miles.
33. $9\frac{3}{11}$. 38. $3\frac{2}{11}$. 42. 8. 46. 400 miles.
47. $10\frac{1}{11}$ minutes past 2 o'clock. 53. $32\frac{1}{11}$ minutes past 6 o'clock.
48. $21\frac{2}{11}$ minutes past 4 o'clock. 54. 300. 55. 600. 56. 700.
49. $32\frac{7}{11}$ minutes past 3 o'clock. 57. Dog, 1440; rabbit, 1800.
50. $43\frac{7}{11}$ minutes past 2 o'clock. 58. \$55,500. 61. \$7400.
51. $5\frac{2}{11}$ minutes past 4 o'clock; 59. \$39,040. 62. 60.
- 38 $\frac{2}{11}$ minutes past 4 o'clock. 60. \$325,000. 63. 80; 81.
52. $38\frac{2}{11}$ minutes past 1 o'clock. 64. 49; 50; 51.
65. 16 gallons of water; 64 gallons of alcohol.
66. Length, 32 feet; breadth, 24 feet.
67. 52; 12. 83. $42\frac{1}{2}$ minutes. 84. 350.
68. 201; 39. 69. $1\frac{1}{2}$. 85. Length, 40 inches; 86. $13\frac{1}{11}$; $7\frac{1}{11}$.
70. $27\frac{1}{11}$ minutes past 2 o'clock. breadth, 21 inches.
71. 36. 73. 30. 87. 8 days. 89. $7\frac{2}{11}$ miles.
72. 61; 30. 74. $1\frac{1}{11}$ days. 88. 63. 90. $40\frac{1}{11}$.
75. 30 days.
76. 144 square feet.
77. 12 miles. 78. 18 miles. 91. A, 14 years; B, 35 years.
79. Hare, 1200; dog, 960. 92. A, \$25; B, \$5.
80. $49\frac{1}{11}$ minutes past 3 o'clock. 93. $32\frac{1}{11}$ minutes past 12 o'clock.
81. 108 miles. 94. \$32,000. 95. $4\frac{1}{2}$ hours.
82. $6\frac{1}{2}$ minutes. 96. $28\frac{1}{2}$ miles.
97. Tin, 37; lead, 23.

98. Gold, 380; silver, 150. 103. \$9804.80.
 99. 4. 101. 3700. 104. A, $2\frac{1}{4}$; B, $3\frac{1}{4}$; C, $4\frac{1}{4}$.
 100. $1\frac{1}{4}$ miles. 102. 70. 105. Hound, 600; fox, 900.

Exercise 68. Page 170.

1. $\frac{abc}{b+c}$ miles. 3. $\frac{b-ac}{c+1}$. 5. $\frac{ab}{b+c}$.
 2. $\frac{ac}{b-a}$. 4. $\frac{abc}{b-a}$ feet. 6. $\frac{d+ac}{b+c}$.
 7. $\frac{a(b-c)}{b-a}$ pieces of 1st kind; $\frac{b(c-a)}{b-a}$ pieces of 2d kind.

Exercise 69. Page 172.

1. 25; 15. 9. 5%. 10. 6%.
 2. $60^{\circ} 45'$; $39^{\circ} 15'$. 11. 3 years 5 months.
 3. $86^{\circ} 23' 7''$; $30^{\circ} 1' 23''$. 12. 20 years. 14. \$1250.
 4. $2\frac{1}{4}$ days. 13. \$537.50. 15. \$40,000.
 5. \$371.25. 7. \$2880. 16. 9 years 4 months 15 days.
 6. \$118.75. 8. 9%. 17. \$20,000. 18. $4\frac{1}{4}\%$.

Exercise 70. Page 177.

1. $x=5$; 4. $x=9$; 7. $x=15$; 10. $x=-2$; 13. $x=9$;
 $y=7$. $y=2$. $y=1$. $y=7$. $y=-3$.
 2. $x=10$; 5. $x=13$; 8. $x=2$; 11. $x=23$; 14. $x=-11$;
 $y=4$. $y=11$. $y=13$. $y=-7$. $y=7$.
 3. $x=3$; 6. $x=19$; 9. $x=5$; 12. $x=15$;
 $y=5$. $y=4$. $y=5$. $y=7$.

Exercise 71. Page 178.

1. $x=7$; 4. $x=9$; 7. $x=3$; 10. $x=10$; 13. $x=-3$;
 $y=2$. $y=10$. $y=7$. $y=7$. $y=-7$.
 2. $x=6$; 5. $x=7$; 8. $x=14$; 11. $x=-4$; 14. $x=-7$;
 $y=4$. $y=9$. $y=7$. $y=8$. $y=2$.
 3. $x=17$; 6. $x=2$; 9. $x=7$; 12. $x=12$;
 $y=11$. $y=1$. $y=-3$. $y=5$.

Exercise 72. Page 179.

- | | | | |
|-----------------------------|-----------------------------|------------------------------|--------------------------------|
| 1. $x = 17$;
$y = 13$. | 5. $x = 31$;
$y = 1$. | 9. $x = 20$;
$y = 13$. | 13. $x = -4$;
$y = -2$. |
| 2. $x = 7$;
$y = 7$. | 6. $x = 13$;
$y = 17$. | 10. $x = 2$;
$y = 11$. | 14. $x = -26$;
$y = 160$. |
| 3. $x = 2$;
$y = 9$. | 7. $x = 5$;
$y = 7$. | 11. $x = 23$;
$y = -1$. | 15. $x = 35$;
$y = 20$. |
| 4. $x = 3$;
$y = 21$. | 8. $x = 7$;
$y = 17$. | 12. $x = 15$;
$y = 8$. | 16. $x = 3$;
$y = 20$. |

Exercise 73. Page 180.

- | | | | | |
|---|-----------------------------|--|------------------------------|-------------------------------|
| 1. $x = 1$;
$y = 2$. | 6. $x = 11$;
$y = 7$. | 11. $x = 3$;
$y = 2$. | 15. $x = 15$;
$y = 60$. | 19. $x = 19$;
$y = 2$. |
| 2. $x = 6$;
$y = 12$. | 7. $x = 2$;
$y = 2$. | 12. $x = 6$;
$y = 5$. | 16. $x = 9$;
$y = 8$. | 20. $x = -3$;
$y = 122$. |
| 3. $x = \frac{1}{4}$;
$y = \frac{1}{5}$. | 8. $x = 13$;
$y = 10$. | 13. $x = 19\frac{1}{2}$;
$y = -17$. | 17. $x = 6$;
$y = 5$. | 21. $x = 11$;
$y = 10$. |
| 4. $x = 9$;
$y = 7$. | 9. $x = 7$;
$y = 10$. | 14. $x = 7$;
$y = 8$. | 18. $x = 3$;
$y = 2$. | 22. $x = 9$;
$y = 7$. |
| 5. $x = 12$;
$y = 6$. | 10. $x = 7$;
$y = 5$. | | | |

Exercise 74. Page 183.

- | | | |
|---|---|---|
| 1. $x = \frac{s+d}{2}$;
$y = \frac{s-d}{2}$. | 4. $x = \frac{m(abn+s)}{ab+c}$;
$y = \frac{m(s-cn)}{ab+c}$. | 7. $x = \frac{2a}{3}$;
$y = \frac{b}{2}$. |
| 2. $x = \frac{n'r - nr'}{mn' - m'n}$;
$y = \frac{m'r - mr'}{m'n - mn'}$. | 5. $x = \frac{abcd}{bd+a}$;
$y = \frac{abc}{bd+a}$. | 8. $x = \frac{a}{5} + b$;
$y = b - \frac{a}{5}$. |
| 3. $x = \frac{b'c + bc'}{ab' + a'b}$;
$y = \frac{ac' - a'c}{ab' + a'b}$. | 6. $x = \frac{a' + a}{a'b + ab'}$;
$y = \frac{b' - b}{ab' + a'b}$. | 9. $x = \frac{a}{b}$;
$y = \frac{b}{c}$. |

10. $x = \frac{c - an - bm}{2b}$; 13. $x = a$;
 $y = \frac{c + an + bm}{2a}$; $y = b$. 16. $x = \frac{a}{a-b}$;
 $y = \frac{b}{a+b}$.
 11. $x = \frac{a}{3}$; $y = \frac{c}{a+b}$. 17. $x = \frac{(m-a)(n-a)}{b-a}$;
 $y = \frac{(m-b)(n-b)}{a-b}$.
 12. $x = a + b$;
 $y = a - b$. 15. $x = \frac{d + 2b^2 - 6a^2}{3a}$;
 $y = \frac{d - b^2 + 3a^2}{3b}$.

Exercise 75. Page 186.

1. $x = 5$;
 $y = 6$. 7. $x = \frac{2}{m+n}$;
 $y = \frac{2}{m-n}$. 10. $x = \frac{ab(a+b)}{c(a^2 + ab + b^2)}$;
 $y = \frac{a+b}{c}$.
 2. $x = \frac{1}{2}$;
 $y = \frac{1}{3}$. 8. $x = \frac{m-n}{b-c}$;
 $y = \frac{n-m}{bn-cm}$. 11. $x = \frac{5}{11a}$;
 $y = \frac{5}{4b}$.
 3. $x = 3$;
 $y = 4$. 9. $x = \frac{ms - nr}{as - bn}$;
 $y = \frac{nr - ms}{ar - bm}$. 12. $x = \frac{1}{b}$;
 $y = \frac{1}{a}$.
 4. $x = \frac{1}{2}$;
 $y = \frac{1}{3}$.
 5. $x = \frac{1}{2}$;
 $y = \frac{1}{4}$.
 6. $x = \frac{1}{2}$;
 $y = \frac{1}{3}$.

Exercise 76. Page 188.

1. $x = -3$;
 $y = 11$;
 $z = 17$. 4. $x = 13$;
 $y = 17$;
 $z = 15$. 7. $x = 7$;
 $y = -3$;
 $z = 1$. 10. $x = 15$;
 $y = \frac{5}{2}$;
 $z = 3$.
 2. $x = 2$;
 $y = 3$;
 $z = 4$. 5. $x = -2$;
 $y = 19$;
 $z = 27$. 8. $x = 5$;
 $y = 5$;
 $z = 5$. 11. $x = 100$;
 $y = 77$;
 $z = -21$.
 3. $x = 4$;
 $y = 2$;
 $z = 2$. 6. $x = 20$;
 $y = 15$;
 $z = -10$. 9. $x = 11$;
 $y = -1$;
 $z = 1$. 12. $x = 10$;
 $y = -12$;
 $z = 14$.

13. $x = 25$; $y = 15$; $z = -7$. 16. $x = \frac{1}{2}$; $y = \frac{1}{2}$; $z = \frac{1}{2}$. 19. $x = \frac{1}{2}$; $y = \frac{1}{2}$; $z = \frac{1}{2}$. 22. $x = \frac{2}{a+b}$; $y = \frac{2}{a+c}$; $z = \frac{2}{b+c}$.
14. $x = -18$; $y = 17$; $z = 16$. 17. $x = -\frac{1}{2}$; $y = 1$; $z = \frac{1}{2}$. 20. $x = \frac{1}{2}$; $y = \frac{1}{2}$; $z = \frac{1}{2}$.
15. $x = \frac{1}{2}$; $y = \frac{1}{2}$; $z = \frac{1}{2}$. 18. $x = 5\frac{1}{2}$; $y = -\frac{1}{2}$; $z = 5$. 21. $x = \frac{1}{2}$; $y = \frac{1}{2}$; $z = \frac{1}{2}$.
24. $x = \frac{2abc}{ac+bc-ab}$; $y = \frac{2abc}{ab+bc-ac}$; $z = \frac{2abc}{ac+ab-bc}$. 25. $x = 24$; $y = 60$; $z = 120$. 27. $x = \frac{1}{2}$; $y = \frac{1}{2}$; $z = \frac{1}{2}$.
26. $x = \frac{7}{2}$; $y = -\frac{7}{2}$; $z = \frac{7}{2}$. 28. $x = \frac{1}{2}$; $y = \frac{1}{2}$; $z = \frac{1}{2}$.

Exercise 77. Page 190.

1. A, \$22; B, \$26. 10. $\frac{1}{2}$; $\frac{1}{2}$. 15. 63.
2. 4.78; 1.0095. 11. $\frac{2}{11}$. 16. 45.
3. A, \$260; B, \$220. 12. $\frac{3}{11}$. 17. 47.
4. 60; 8. 5. 5.37; 1.25. 13. $\frac{8}{15}$. 18. 640.
6. A, \$31; B, \$27. 8. $\frac{7}{19}$. 14. 876. 19. 243.
7. $\frac{5}{11}$. 9. $\frac{1}{2}$. 20. 3 hours down; 5 hours up.
21. Crew, $7\frac{1}{2}$ miles per hour; current, $2\frac{1}{2}$ miles per hour.
22. $11\frac{1}{2}$ miles; 5 miles per hour.
23. 4 hours down; 6 hours up; $\frac{1}{2}$ mile per hour.
24. A, $5\frac{7}{13}$ days; B, $6\frac{6}{11}$ days; C, $14\frac{2}{3}$ days.
25. A, 1 hour 45 minutes; B, 3 hours 30 minutes; C, 7 hours.
26. A, $4\frac{1}{2}$ days; B, $5\frac{1}{2}$ days; C, $13\frac{1}{2}$ days.
27. A, $\frac{2abc}{bc+ac-ab}$ days; B, $\frac{2abc}{bc-ac+ab}$ days; C, $\frac{2abc}{ac+ab-bc}$ days.
28. \$2000; 6%. 30. \$2000; 6%. 31. \$6000 at 5%; \$4000 at 6%.
29. \$25,000; 4%. 32. A, $5\frac{1}{2}$; B, $5\frac{1}{2}$.
33. A, $6\frac{2}{3}$ yards a second; B, $6\frac{1}{3}$ yards a second; A, $12\frac{2}{3}$ miles an hour; B, $12\frac{1}{2}$ miles an hour.
34. A, $7\frac{1}{2}$ yards a second; B, $7\frac{1}{2}$ yards a second; A, $15\frac{1}{2}$ miles an hour; B, $15\frac{1}{2}$ miles an hour.
35. 30 miles an hour. 38. Wheat, \$1.00; rye, \$0.75; oats, \$0.33.
36. 50 miles an hour. 39. 900 miles.
37. Velvet, \$12; silk, \$2.

40. 15 persons; \$75. 42. 2.15 P.M.; $18\frac{1}{2}$ miles.
 41. Cow, \$50; sheep, \$5. 43. 759.
 44. Length, 35 yards; breadth, 10 yards.
 45. Length, 100 feet; breadth, 60 feet.
 46. Length, 16 feet; breadth, 9 feet. 47. 36.
 48. First person, \$5000 at 4%; second, \$7000 at 5%; third, \$8000 at 6%.
 49. \$10,000 at 4%; \$7000 at $4\frac{1}{2}\%$. 50. 48.
 51. \$3000 at $4\frac{1}{2}\%$; \$4000 at 5%.
 52. $\frac{dm - cn}{m - n}$ dollars; $\frac{100(c - d)}{dm - cn} \%$
 53. $\frac{bm - an}{m - n}$ dollars; $\frac{1200(a - b)}{bm - an} \%$
 54. \$9000 in 5% bonds; \$9375 in 3%.
 55. A, 5 yards a second; B, $4\frac{1}{2}$ yards a second.
 56. 30 miles an hour.
 57. 30 at 3 for 5 cents; 40 at 4 for 5 cents. 58. 750 square feet.
 59. 7 miles an hour; $10\frac{1}{2}$ miles an hour.
 60. 5 quarts from first box; 2 quarts from second box.
 61. 63 miles. 62. 480 at 16 for 25 cents; 90 at 18 for 25 cents.
 63. A, 5 minutes; B, 5 minutes 20 seconds.

Exercise 78. Page 204.

1. $\frac{bm}{a - b}$ hours. 3. $\frac{(m - b)l}{a - b}$ gallons of first kind;
 2. $\frac{ab}{b - ac}$ miles per hour. $\frac{(a - m)l}{a - b}$ gallons of second kind.

Exercise 79. Page 207.

- | | | |
|--------------------------------|--|-----------------------------|
| 1. $x = 10, 8;$
$y = 1, 3.$ | 6. $x = 4, 9;$
$y = 13, 5.$ | 11. $x = 21;$
$y = 12.$ |
| 2. $x = 1, 4;$
$y = 11, 4.$ | 7. $x = 6;$
$y = 2.$ | 12. $x = 49;$
$y = 22.$ |
| 3. $x = 5;$
$y = 4.$ | 8. $x = 28, 18, 8;$
$y = 1, 8, 15.$ | 13. $x = 32;$
$y = 21.$ |
| 4. $x = 19, 9;$
$y = 1, 2.$ | 9. $x = 3;$
$y = 5.$ | 14. $x = 19;$
$y = 476.$ |
| 5. $x = 15, 7;$
$y = 2, 5.$ | 10. $x = 8;$
$y = 1.$ | |

Exercise 88. Page 229.

- | | | | |
|---------|------------|-------------|-------------|
| 1. 17. | 5. 72.8. | 9. 1.4454. | 13. 0.8735. |
| 2. 35. | 6. 12.25. | 10. 1.2703. | 14. 0.9085. |
| 3. 112. | 7. 4.4310. | 11. 0.3684. | 15. 0.9352. |
| 4. 503. | 8. 2.1544. | 12. 0.8780. | 16. 0.4252. |

Exercise 89. Page 229.

- | | | | |
|--------------|-------------------|--------------|-------------|
| 1. $3x + 1.$ | 3. $1 + x + x^2.$ | 5. $3 - x.$ | 7. $1 - y.$ |
| 2. $2x - a.$ | 4. $1 + d.$ | 6. $1 - 3y.$ | |

Exercise 90. Page 234.

- | | | |
|--|--|--|
| 1. $a^{\frac{2}{3}}.$ | 18. $\frac{1}{a^{\frac{2}{3}}}.$ | 34. 216. |
| 2. $a^{\frac{4}{3}}.$ | 19. $\frac{3y^3}{x^2}.$ | 35. - 243. |
| 3. $a^{\frac{7}{3}}.$ | 20. $\frac{4x}{y^7}.$ | 36. - 9375. |
| 4. $(-8)^{\frac{1}{3}}.$ | 21. $\frac{4}{x^3y^2}.$ | 37. $7\frac{1}{3}.$ |
| 5. $a^{\frac{1}{3}}.$ | 22. $\frac{3b^{\frac{2}{3}}}{a^4}.$ | 38. 2. |
| 6. $a^{\frac{5}{3}}.$ | 23. $\frac{3^2 \times 2x^2y^5}{ab^2}.$ | 39. $\frac{1}{4b}.$ |
| 7. $a^{\frac{1}{3}} + x^{\frac{1}{3}} + 2b^{\frac{1}{3}}.$ | 24. $3x^5y^2z^3.$ | 40. $\frac{1}{2}b.$ |
| 8. $a^{\frac{2}{3}}x + ac^{\frac{1}{3}}.$ | 25. $x^5z^3.$ | 41. $\frac{b}{\sqrt{a}}.$ |
| 9. $\sqrt[4]{a^8}.$ | 26. $a^3c^3d^{-1}.$ | 42. $\frac{a}{b^{\frac{2}{3}}}.$ |
| 10. $\sqrt[3]{c^3}.$ | 27. $ac.$ | 43. $ab^3.$ |
| 11. $\sqrt{a} \sqrt[3]{b}.$ | 28. $a^2b^2x^{-2}y^{\frac{2}{3}}.$ | 44. $x^{-\frac{1}{3}}y^{\frac{1}{3}}z^{-\frac{1}{6}}.$ |
| 12. $\sqrt[3]{ab^2}.$ | 29. $a^3c^{-3}.$ | 45. 1. |
| 13. $\sqrt[3]{\frac{x}{y}}.$ | 30. 32. | 46. 1. |
| 14. $\frac{3\sqrt[3]{x^2}}{\sqrt[4]{y^3}}.$ | 31. $\frac{1}{12}.$ | 47. 2. |
| 15. $\sqrt{a} - \sqrt[3]{cx}.$ | 32. 9. | 48. $\frac{1}{2}.$ |
| 16. $\sqrt[3]{a^2} + \sqrt[3]{c^2x^2}.$ | 33. $\frac{1}{4}.$ | 49. 6. |
| 17. $\frac{1}{a^3}.$ | | 50. 1. |
| | | 51. 2. |
| | | 52. $\frac{1}{4}.$ |

Exercise 91. Page 236.

1. $a - b$.
2. $a + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b$.
3. $a - 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b$.
4. $x^3 - 4x^2$.
5. $x^{-4} + x^{-2}y^{-2} + y^{-4}$.
6. $x + y$.
7. $x - y$.
8. $1 + b^{-2} + b^{-4}$.
9. $2a^{\frac{2}{3}}b^{-1} - 20 + 18a^{-\frac{1}{3}}b$.

Exercise 92. Page 237.

1. $a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}$.
2. $a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}$.
3. $a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{2}{3}}$.
4. $a^{\frac{4}{3}} - a^{\frac{2}{3}}b^{\frac{2}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}$.
5. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1$.
6. $x^{\frac{1}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{1}{3}}$.
7. $x^{\frac{1}{3}} - 2 - 3x^{-\frac{1}{3}}$.
8. $3x^{\frac{1}{3}} - 2 - x^{-\frac{1}{3}}$.
9. $x^{-3} + 2x^{-2}y^{-1} - 4x^{-1}y^{-2}$.
10. $x^{\frac{2}{3}} - x^{\frac{1}{3}}(y^{\frac{1}{3}} + z^{\frac{1}{3}}) + (y^{\frac{2}{3}} - y^{\frac{1}{3}}z^{\frac{1}{3}} + z^{\frac{2}{3}})$.

Exercise 93. Page 237.

1. $x^{\frac{1}{3}} + 1$.
2. $2a^{\frac{1}{3}} - b^{\frac{1}{3}}$.
3. $x^{\frac{1}{3}} - 2$.
4. $2a^{-1} + 1$.
5. $3a^{\frac{1}{3}} - 2 + a^{-\frac{1}{3}}$.
6. $m + 1 - m^{-1}$.
7. $7x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1$.

Exercise 94. Page 239.

1. $\sqrt{5}$.
2. $\sqrt{2}$.
3. $\sqrt{3}$.
4. $\sqrt{7}$.
5. 2.
6. $\sqrt[3]{ab}$.
7. $\sqrt{ab}$.
8. $\sqrt{ab}$.
9. $b\sqrt{3a}$.
10. $\sqrt{2ab}$.
11. $\frac{1}{2}\sqrt[3]{\frac{5a}{b}}$.
12. $2\sqrt{\frac{x}{x-3}}$.
13. $b\sqrt{\frac{a}{2xy}}$.

Exercise 95. Page 240.

1. $2\sqrt{7}$.
2. $6\sqrt{2}$.
3. $2\sqrt[3]{9}$.
4. $5\sqrt[3]{4}$.
5. $6\sqrt[3]{2}$.
6. $4\sqrt[3]{3}$.
7. $2\sqrt[5]{4}$.
8. $3\sqrt[4]{3}$.
9. $2\sqrt[4]{11}$.
10. $9\sqrt{5}$.
11. $4\sqrt[4]{7}$.
12. $18\sqrt[5]{4}$.
13. $14\sqrt{3}$.
14. $8m\sqrt{n}$.
15. $3b^2\sqrt[4]{a^3}$.
16. $2a^3\sqrt[5]{a^2c^4}$.
17. $11a^2b^2\sqrt{b}$.
18. $14a^3\sqrt{b}$.
19. $18n^3\sqrt{m^2}$.
20. $4xy^2\sqrt{x^3}$.
21. $7\sqrt[3]{3}$.
22. $-9\sqrt[3]{3}$.
23. $5\sqrt[4]{2}$.
24. $24\sqrt[3]{3}$.
25. $\frac{4x^2}{3mn}\sqrt[3]{y}$.
26. $\frac{2a}{3}\sqrt{ab}$.
27. $\frac{5x}{6y}$.
28. $\frac{2m}{3}\sqrt[5]{mn}$.
29. $\frac{xy}{6}\sqrt[4]{xy^3}$.
30. $\frac{(x-y)z}{8}$.
31. $\sqrt{5}$.

Exercise 96. Page 241.

- | | | | |
|----------------------------------|-------------------------------------|---|---------------------------------|
| 1. $\sqrt{2}$. | 4. $\frac{1}{5}\sqrt{5}$. | 7. $\frac{2}{3}\sqrt[3]{9}$. | 10. $\frac{2}{3}\sqrt[3]{45}$. |
| 2. $\frac{1}{4}\sqrt{6}$. | 5. $\frac{1}{2}\sqrt{5}$. | 8. $\frac{1}{3}\sqrt[3]{9}$. | 11. $\sqrt[5]{6}$. |
| 3. $\frac{1}{10}\sqrt{5}$. | 6. $\frac{2}{10}\sqrt{5}$. | 9. $\frac{2}{3}\sqrt[3]{9}$. | 12. $\sqrt[5]{6}$. |
| 13. $\frac{a^2}{b^2}\sqrt{bc}$. | 15. $\frac{x}{b}\sqrt[3]{abx}$. | 17. $\frac{ay}{bd}\sqrt{bc}$. | |
| 14. $\frac{b}{a}\sqrt[4]{a}$. | 16. $\frac{1}{5x}\sqrt[3]{7ax^2}$. | 18. $\frac{2a}{3xyz}\sqrt[3]{18b^2cxy^2}$. | |

Exercise 97. Page 242.

- | | | | |
|---------------------|----------------------|--------------------------|------------------------------------|
| 1. $\sqrt{125}$. | 5. $\sqrt[4]{48}$. | 9. $\sqrt[3]{-8y}$. | 13. $\sqrt{\frac{a}{4}}$. |
| 2. $\sqrt{99}$. | 6. $\sqrt[5]{486}$. | 10. $-\sqrt{9y^3}$. | 14. $\sqrt[3]{-\frac{8a^2}{27}}$. |
| 3. $\sqrt[3]{81}$. | 7. $\sqrt[5]{128}$. | 11. $-\sqrt[5]{10m^5}$. | 15. $\sqrt{\frac{9m^8}{16}}$. |
| 4. $\sqrt[3]{32}$. | 8. $\sqrt{8}$. | 12. $-\sqrt[4]{16x}$. | 16. $-\sqrt[4]{\frac{m^7}{16}}$. |

Exercise 98. Page 243.

- | | |
|--|--|
| 1. $\sqrt[12]{27}$; $\sqrt[12]{25}$. | 7. $\sqrt[12]{64}$; $\sqrt[12]{81}$; $\sqrt[12]{125}$. |
| 2. $\sqrt[5]{196}$; $\sqrt[5]{216}$. | 8. $\sqrt[5]{a^2}$; $\sqrt[5]{b^2}$; $\sqrt[5]{c^3}$. |
| 3. $\sqrt[5]{8}$; $\sqrt[5]{16}$. | 9. $\sqrt[10]{a^8}$; $\sqrt[10]{c^3}$; $\sqrt[10]{x^{15}}$. |
| 4. $\sqrt[5]{a^3}$; $\sqrt[5]{b^4}$. | 10. $\sqrt[12]{x^6y^3}$; $\sqrt[12]{a^4b^4c^4}$; $\sqrt[12]{4z^2}$. |
| 5. $\sqrt[5]{125}$; $\sqrt[5]{75}$. | 11. $\sqrt[12]{(x-y)^2}$; $\sqrt[12]{(x+y)^3}$. |
| 6. $\sqrt[12]{64}$; $\sqrt[12]{512}$; $\sqrt[12]{256}$. | 12. $\sqrt[5]{(a+b)^2}$; $\sqrt[5]{(a-b)^3}$. |
13. Ascending order of magnitude, $\sqrt{6}$; $\sqrt[3]{15}$.
14. Ascending order of magnitude, $\sqrt[3]{4}$; $\sqrt{3}$.
15. Ascending order of magnitude, $\sqrt[5]{80}$; $\sqrt[3]{9}$; $\sqrt{8}$.
16. Ascending order of magnitude, $\sqrt[4]{7}$; $\sqrt[3]{5}$; $\sqrt{3}$.

Exercise 99. Page 244.

- | | | |
|-----------------------------------|--|----------------------------|
| 1. $2\sqrt{11}$. | 12. $5\sqrt{b}$. | 23. $(a-b)^2\sqrt{x}$. |
| 2. $6\sqrt{3}$. | 13. $11\sqrt{7}$. | 24. $(x+y)^2\sqrt{z}$. |
| 3. $6\sqrt[3]{4}$. | 14. $-\sqrt{2}$. | 25. $(2ab-2a)\sqrt{c}$. |
| 4. $5\sqrt[5]{2}$. | 15. $12\sqrt{3}$. | 26. $(7a-2)\sqrt[4]{a}$. |
| 5. $\frac{7}{3}\sqrt[3]{5}$. | 16. $129\sqrt{5}$. | 27. $(3m+1)\sqrt[3]{m}$. |
| 6. $-4\sqrt[4]{3}$. | 17. $19\sqrt{5}-23\sqrt{3}$. | 28. $-(7+5a)\sqrt{2a}$. |
| 7. $12\sqrt{3}$. | 18. $6\sqrt{5}$. | 29. $20ab\sqrt{7ab}$. |
| 8. $51\sqrt{3}$. | 19. $\frac{1}{4}\sqrt{2}-2\sqrt[3]{2}$. | 30. $7mn\sqrt{5m}$. |
| 9. $3\sqrt{a}$. | 20. $2\sqrt{15}$. | 31. $(2ab+6)^2\sqrt{2b}$. |
| 10. $-\frac{1}{2}\sqrt[3]{a^2}$. | 21. $(c+5)\sqrt[3]{c}$. | 32. $5a^2\sqrt[3]{b}$. |
| 11. $(a+b-3)\sqrt{a}$. | 22. $(a-b+2)\sqrt[5]{b}$. | |

Exercise 100. Page 246.

- | | | | | |
|--------|----------------------|--------------------------------|------------------------|---------------------------------|
| 1. 9. | 7. 2. | 13. $3\sqrt[5]{2}$. | 19. 18. | 25. $6\sqrt[5]{3}$. |
| 2. 10. | 8. 3. | 14. 8. | 20. 4. | 26. $\frac{1}{3}\sqrt[5]{2}$. |
| 3. 6. | 9. $2\sqrt{6}$. | 15. -2. | 21. $-\frac{5}{4}$. | 27. $\sqrt[5]{\frac{7}{192}}$. |
| 4. 3. | 10. $3\sqrt{2}$. | 16. -7. | 22. $\sqrt[5]{2000}$. | 28. $3\sqrt[5]{243}$. |
| 5. 4. | 11. $3\sqrt[3]{2}$. | 17. $-9\sqrt[3]{5}$. | 23. $2\sqrt{10}$. | 29. $\sqrt[5]{\frac{5}{16}}$. |
| 6. 3. | 12. $2\sqrt[4]{3}$. | 18. $\frac{3}{5}\sqrt[3]{2}$. | 24. 4. | 30. $\sqrt[19]{\frac{1}{54}}$. |

Exercise 101. Page 247.

- | | | |
|---|--|-----------------------|
| 1. $\sqrt{21}$. | 3. $-4+\sqrt{5}$. | 5. $-15-19\sqrt{3}$. |
| 2. 8. | 4. $10-2\sqrt{2}$. | 6. $36-15\sqrt{6}$. |
| 7. $6\sqrt{2}-3\sqrt{15}+8\sqrt{3}-6\sqrt{10}$. | 10. 30. | |
| 8. $7\sqrt{2}-\sqrt{6}+7\sqrt{5}-\sqrt{15}$. | 11. $54-7\sqrt{35}-26\sqrt{21}-7\sqrt{15}$. | |
| 9. $8-8\sqrt[3]{12}+\sqrt[3]{18}$. | 12. $140+10\sqrt{6}$. | |
| 13. $-1+3\sqrt[3]{2}-2\sqrt[3]{3}-3\sqrt[3]{4}+\sqrt[3]{12}+4\sqrt[3]{9}+2\sqrt[3]{18}$. | | |
| 14. $-\sqrt{2}$. | 15. $176+66\sqrt{5}$. | 16. 243. |

Exercise 102. Page 248.

- | | | |
|--------------------|----------------------------|-------------------------------|
| 1. 9. | 5. $\frac{5}{8}\sqrt{2}$. | 9. $\frac{17}{8}$. |
| 2. 3. | 6. $\frac{7}{8}$. | 10. $\sqrt{2}+5\sqrt{6}$. |
| 3. $a^2\sqrt{3}$. | 7. $\frac{7}{8}$. | 11. $7\sqrt{3}-3\sqrt{5}$. |
| 4. $\frac{3}{8}$. | 8. $\frac{11}{8}$. | 12. $4\sqrt{35}+8\sqrt{14}$. |

13. $2\sqrt[3]{25} - 6\sqrt[3]{4} + 5\sqrt[3]{36}$. 14. $5\sqrt[3]{75} + 3\sqrt[3]{18} - 4\sqrt[3]{36}$.
 15. 3. 19. $\frac{1}{3}\sqrt{10}$. 21. 0.2.
 16. $\sqrt[3]{7}$. 20. $\frac{1}{x^2}\sqrt[3]{4x^5}$. 22. $\frac{1}{x+y}\sqrt{x^2-y^2}$.
 17. $\frac{1}{3}\sqrt[3]{486}$.
 18. $\frac{4}{15}$.

Exercise 103. Page 249.

1. $\frac{1}{b}\sqrt{b} + a\sqrt{a}$. 4. $\frac{19-6\sqrt{2}}{17}$. 7. $-\frac{92+15\sqrt{7}}{83}$.
 2. $\frac{1}{15}\sqrt{13}$. 5. $\frac{3\sqrt{7}-2\sqrt{3}}{3}$. 8. $\frac{84+21\sqrt{6}}{10}$.
 3. $\frac{33-9\sqrt{7}}{58}$. 6. $\sqrt{3}-\sqrt{2}$. 9. $\frac{24\sqrt{7}-21\sqrt{2}}{2}$.
 10. $4-\sqrt{15}$. 11. $\frac{11+3\sqrt{14}}{5}$. 12. $\frac{28\sqrt{5}+15\sqrt{10}-35-60\sqrt{2}}{55}$.
 13. 7.0710... 16. 0.0447... 19. 0.2357... 22. 0.0209...
 14. 4.6188... 17. 0.0641... 20. 0.0894... 23. 22.1803..
 15. 5.3665... 18. 0.2886... 21. 0.1118... 24. 0.6185...

Exercise 104. Page 250.

1. m^2 . 4. y^{20} . 7. $4a^6b^2$. 10. $\sqrt[3]{a}$. 13. $\sqrt[3]{(3a-2b)^2}$.
 2. $m^7\sqrt{m}$. 5. $x-y$. 8. $(x^2-y^2)^2$. 11. $\sqrt{3}$. 14. $a^3\sqrt[3]{2}$.
 3. x^{12} . 6. $(a-b)^2$. 9. $\sqrt{x}$. 12. $\sqrt{5}$. 15. $2a^2\sqrt[3]{3}$.

Exercise 105. Page 254.

1. $2-\sqrt{3}$. 7. $\frac{5\sqrt{2}+\sqrt{14}}{2}$. 13. $7+3\sqrt{5}$.
 2. $3+\sqrt{2}$. 8. $3\sqrt{7}+2\sqrt{3}$. 14. $\sqrt{6}-\sqrt{5}$.
 3. $\sqrt{2}+\sqrt{5}$. 9. $4+\sqrt{3}$. 15. $2\sqrt{11}-\sqrt{3}$.
 4. $\sqrt{10}+2\sqrt{2}$. 10. $2\sqrt{2}+2\sqrt{3}$. 16. $3\sqrt{2}+\sqrt{11}$.
 5. $\sqrt{3}+\sqrt{5}$. 11. $4-2\sqrt{3}$. 17. $3\sqrt{7}+2\sqrt{5}$.
 6. $\sqrt{7}-2\sqrt{2}$. 12. $3\sqrt{3}+2\sqrt{6}$. 18. $2\sqrt{7}-3\sqrt{3}$.

Exercise 106. Page 255.

- | | | | |
|------------------------|------------------------|------------------------|-------------------------|
| 1. $x = 2.$ | 8. $x = \frac{2}{3}.$ | 15. $x = 5.$ | 22. $x = 4.$ |
| 2. $x = 3.$ | 9. $x = -6.$ | 16. $x = 49.$ | 23. $x = 4a - 4$ |
| 3. $x = \frac{7}{2}.$ | 10. $y = 5.$ | 17. $x = 4.$ | 24. $x = \frac{81}{a}.$ |
| 4. $x = \frac{4}{3}.$ | 11. $x = \frac{5}{2}.$ | 18. $x = 8.$ | 25. $x = 4a.$ |
| 5. $y = \frac{1}{3}.$ | 12. $x = \frac{1}{2}.$ | 19. $x = \frac{1}{3}.$ | 26. $x = 8.$ |
| 6. $x = -\frac{1}{3}.$ | 13. $x = 49.$ | 20. $x = \frac{2}{3}.$ | |
| 7. $x = -12.$ | 14. $x = 5.$ | 21. $x = \frac{1}{2}.$ | |

Exercise 107. Page 260.

- | | | | |
|--|--|---|---------------------------------|
| 1. $3\sqrt{-1}.$ | 6. $x\sqrt{-1}.$ | 11. $2\sqrt{-1}.$ | 16. $x^{\frac{2}{3}}\sqrt{-1}.$ |
| 2. $4\sqrt{-1}.$ | 7. $9\sqrt{-1}.$ | 12. $3\sqrt{-1}.$ | 17. $x^3\sqrt{-1}.$ |
| 3. $5\sqrt{-1}.$ | 8. $16\sqrt{-1}.$ | 13. $17\sqrt{-1}.$ | 18. $\frac{1}{2}\sqrt{-1}.$ |
| 4. $12\sqrt{-1}.$ | 9. $25\sqrt{-1}.$ | 14. $2\sqrt{-1}.$ | 19. $a^2b^{-1}\sqrt{-1}.$ |
| 5. $13\sqrt{-1}.$ | 10. $6\sqrt{-1}.$ | 15. $x^4\sqrt{-1}.$ | 20. $3x^2\sqrt{-1}.$ |
| 21. $(2x-3y)^2\sqrt{-1}.$ | 24. $(x^2-y^2)^{\frac{1}{2}}\sqrt{-1}.$ | 27. $7a^2\sqrt{-1}.$ | |
| 22. $(x-2y)^2\sqrt{-1}.$ | 25. $\sqrt{-1}.$ | 28. $9a\sqrt{-1}.$ | |
| 23. $(x^2+y^2)^{\frac{1}{2}}\sqrt{-1}.$ | 26. $3\sqrt{-1}.$ | 29. $2a.$ | 30. $6\sqrt{-1}$ |
| 31. $a+c+(b-d)\sqrt{-1}.$ | 39. $-40.$ | 41. $-6\sqrt{6}.$ | |
| 32. $(a+b)\sqrt{-1}.$ | 40. $-\sqrt{a^2-b^2}.$ | 42. $10\sqrt{10}.$ | |
| 33. $-\sqrt{15}.$ | 36. $-x^{\frac{3}{2}}.$ | 43. $-2\sqrt{2}+\sqrt{10}-2\sqrt{3}+\sqrt{15}.$ | |
| 34. $5.$ | 37. $-xy.$ | 44. $x^2-x+1.$ | 45. $b^3-a^3.$ |
| 35. $-12.$ | 38. $-8\sqrt{2}.$ | 46. $-12\sqrt{2}+4\sqrt{10}-18\sqrt{3}+6\sqrt{15}.$ | 47. $15.$ |
| 48. $mn+4m\sqrt{-c}-3n\sqrt{-b}+12\sqrt{bc}.$ | | | |
| 49. $-a\sqrt{-1}.$ | 52. $\frac{1}{3}.$ | 55. $-\sqrt{-x}.$ | 59. $x\sqrt{2}.$ |
| 50. $-\sqrt{-1}.$ | 53. $\frac{\sqrt{a}}{\sqrt{b}}.$ | 56. $-\sqrt{-1}.$ | 60. $4\sqrt{-x}.$ |
| 51. $-\frac{c\sqrt{-1}}{2}.$ | 54. $\sqrt{a}.$ | 57. $-\sqrt{x}.$ | 61. $\frac{3+\sqrt{-2}}{11}.$ |
| 62. $\frac{2+2\sqrt{-1}+\sqrt{-2}-\sqrt{2}}{2}.$ | 63. $\frac{a^2-x^2+2ax\sqrt{-1}}{a^2+x^2}.$ | | |
| 64. $\frac{\sqrt{30}+2\sqrt{-10}+6\sqrt{-1}-4\sqrt{3}}{14}.$ | | | |
| 65. $\frac{4a^2-9b^2+12ab\sqrt{-1}}{4a^2+9b^2}.$ | 66. $\frac{8a^3+8b^3-63ab\sqrt{-1}}{64a^2+b^2}.$ | | |

21. $x = b - 2a$, or $b - 2a$.

22. $x = b$, or $-a$.

23. $x = a$, or $-\frac{a}{7}$.

24. $x = 3a$, or $\frac{13}{7}a$.

25. $x = \frac{3b - 2a}{2}$, or $b - a$.

26. $x = a - b$, or $-a - 2b$.

27. $x = -a$, or $-b$.

28. $x = \frac{a}{3}$, or $\frac{2b}{3}$.

29. $x = \frac{a - a^2}{2a + 1}$, or $-a$.

30. $x = \frac{1 + a}{1 - a}$, or $\frac{1 - a}{1 + a}$.

31. $x = \frac{a}{a + b}$, or $-\frac{b}{a + b}$.

42. $x = \frac{a^2 + b^2 \pm \sqrt{(a^2 - b^2)^2 + 4ab^2}}{2ab}$.

43. $x = \pm 1$.

44. $x = \pm m\sqrt{\frac{c - 2}{c}}$.

45. $x = \frac{a + b}{a - b}$, or $\frac{a - b}{a + b}$.

46. $x = (m + n)^2$, or $-(m - n)^2$.

47. $x = 2a$, or $\frac{a + b}{2}$.

48. $x = m \pm (n - p)$.

49. $x = \frac{m + n \pm (p + q)}{2}$.

32. $x = 1$, or $\frac{a}{a + b}$.

33. $x = a$, or $-\frac{b(a + b)}{2a + b}$.

34. $x = 1 \pm \sqrt{-2}$.

35. $x = a^2 + b^2 \pm (a - b)\sqrt{a^2 + b^2}$.

36. $x = \frac{a + 2b}{2}$, or $\frac{a - b}{2}$.

37. $x = \frac{1 - 3a \pm 2a\sqrt{a}}{(a - 1)^2}$.

38. $x = 0$, or $\frac{2ab - ac - bc}{a + b - 2c}$.

39. $x = \frac{c \pm \sqrt{4ac + c^2}}{2(a + b)}$.

40. $x = \frac{6 \pm 3\sqrt{a^2 + 4}}{a}$.

41. $x = \frac{m}{n}$, or $-\frac{n}{m^2}$.

50. $x = m + n$, or $\frac{m + n}{mn}$.

51. $x = a + \frac{m}{2}$, or $m - 2a$.

52. $x = \frac{3a - 2b}{6}$, or $\frac{2a - 3b}{6}$.

53. $x = 2a - b$, or $a + 2b$.

54. $x = 2a + 6b$, or $a - 8b$.

55. $x = \frac{a - 6b}{2}$, or $-(5a + b)$.

56. $x = \frac{4a^2 + 9b^2}{2a + 3b}$, or $-(2a + 3b)$.

Exercise 115. Page 278.

1. $x = 2$, or $-\frac{7}{3}$.

5. $x = 2$, or $-\frac{5}{6}$.

9. $x = 1$, or $-\frac{13}{8}$.

2. $x = 3$, or $-\frac{4}{3}$.

6. $x = 3$, or $\frac{2}{3}$.

10. $x = \frac{2}{3}$, or $\frac{5}{33}$.

3. $x = 9$, or -2 .

7. $x = 1$, or $\frac{2}{3}$.

11. $x = 2$, or $-\frac{13}{7}$.

4. $x = 3$, or $-\frac{14}{5}$.

8. $x = 4$, or $-\frac{7}{4}$.

12. $x = 2$, or $-\frac{3}{5}$.

Exercise 116. Page 280.

- | | | |
|--|-----------------------------------|---------------------|
| 1. $x = 9$. | 7. $x = 0$, or 8. | 13. $x = \pm 2$. |
| 2. $x = \pm \frac{1}{2}\sqrt{4ab-b^2}$. | 8. $x = 0$, or 3. | 14. $x = 6$. |
| 3. $x = 0$, or 2. | 9. $x = 6$. | 15. $x = 12$. |
| 4. $x = 39$. | 10. $x = \frac{4}{3}$, or -2 . | 16. $x = 2$, or 1. |
| 5. $x = \frac{1}{2}$, or $-\frac{2}{3}$. | 11. $x = 2$, or -1 . | 17. $x = 3$. |
| 6. $x = 4$. | 12. $x = 5$, or 1. | |

Exercise 117. Page 283.

- | | |
|--|--|
| 1. $x = \pm 2$, or ± 1 . | 17. $x = 3, -\frac{2}{3}$, or $\frac{-3 \pm \sqrt{-55}}{4}$. |
| 2. $x = \pm 3$, or ± 2 . | 18. $x = \frac{1}{3}, -\frac{1}{3}, 0$, or -5 . |
| 3. $x = \pm 5$, or $\pm 2\sqrt{-1}$. | 19. $x = 2, -\frac{1}{2}$, or $\frac{3 \pm \sqrt{505}}{4}$. |
| 4. $x = \sqrt[3]{3}$, or $\frac{1}{2}\sqrt[3]{18}$. | 20. $x = \frac{7}{2}, -\frac{3}{2}$, or $1 \pm \sqrt{5}$. |
| 5. $x = \pm \frac{3}{2}$, or $\pm \frac{1}{2}\sqrt{-19}$. | 21. $x = 3, -\frac{5}{3}$, or $\frac{2 \pm \sqrt{70}}{3}$. |
| 6. $x = \pm \frac{1}{2}\sqrt{6}$, or $\pm \frac{1}{2}\sqrt{-35}$. | 22. $x = 5, \frac{1}{3}$, or $\frac{8 \pm 2\sqrt{37}}{3}$. |
| 7. $x = \frac{1}{3}$, or $-\frac{243}{8}$. | 23. $x = 4, -3, 2$, or -1 . |
| 8. $x = 256$, or $\frac{525}{16}$. | 24. $x = 5, -3, 1$, or 1. |
| 9. $x = \sqrt[3]{\frac{9}{10}}$, or $\sqrt[3]{-\frac{3}{2}}$. | 25. $x = 3, -\frac{1}{2}, 2$, or $\frac{1}{2}$. |
| 10. $x = \frac{1}{2}$, or $-\frac{3}{2}$. | 26. $x = \frac{5}{2}, -1$, or $\frac{3}{4} \pm \frac{1}{4}\sqrt{-15}$. |
| 11. $x = 0, 1$, or $\frac{1}{2} \pm \frac{1}{2}\sqrt{-2}$. | 27. $x = 2, -\frac{1}{2}$, or $\frac{3 \pm \sqrt{-39}}{4}$. |
| 12. $x = 2$, or 1. | |
| 13. $x = \frac{2}{3}$, or $\frac{4}{3}$. | |
| 14. $x = 3$, or 8. | |
| 15. $x = \pm 3$, or ± 2 . | |
| 16. $x = 2, -4$, or $-1 \pm \frac{1}{2}\sqrt{114}$. | |
| 28. $x = \frac{1}{2}(-1 \pm \sqrt{5})$, or $\frac{1}{2}(-1 \pm \frac{1}{2}\sqrt{58})$. | |

Exercise 118. Page 286.

- The roots are real, unequal, and rational.
- The roots are real, unequal, and rational.
- The roots are imaginary.
- The roots are real, unequal, and rational.
- The roots are real, equal, and rational.
- The roots are real, unequal, and rational.
- The roots are real, unequal surds.
- The roots are imaginary.
- The roots are real, unequal, and rational.
- The roots are real, equal, and rational.

11. $n = -\frac{1}{2}$, or -3 .

12. $m = 2$, or $\frac{2}{3}$.

13. $m = 4$, or $-\frac{1}{2}$.

14. $m = \frac{11 \pm 4\sqrt{22}}{7}$.

15. $m = \frac{-1 \pm 12\sqrt{-19}}{17}$.

Exercise 119. Page 287.

1. $x^2 - 13x + 42 = 0$.

2. $x^2 - 2x - 15 = 0$.

3. $2x^2 + x - 6 = 0$.

4. $3x^2 - 19x + 28 = 0$.

5. $6x^2 - x - 12 = 0$.

6. $18x^2 + 51x + 35 = 0$.

7. $3x^2 - 26x - 169 = 0$.

8. $121x^2 - 55x + 6 = 0$.

9. $x^2 - 6x + 7 = 0$.

10. $x^2 - 2x + 2 = 0$.

11. $x^2 - 2ax + bx + a^2 - ab = 0$.

12. $x^2 - 2ax + a^2 - b^2 = 0$.

Exercise 120. Page 290.

1. 19; 20.

3. 12; 14.

5. 5; 6; 7.

2. 3; 4.

4. 5; 6.

6. 9; 11.

7. Length, 6 rods; breadth, 4 rods.

8. 15 feet.

9. $1\frac{1}{2}$ yards.

10. Length, 20 rods; breadth, 10 rods.

11. Length, 24 rods; breadth, 7 rods.

12. $\frac{2}{3}$.

13. $\frac{2}{3}$.

19. \$90.

25. 9 feet.

14. \$6.

20. 20.

26. 10 miles per hour.

15. 18.

21. Larger, 10 hours; smaller, 14 hours.

16. 12.

22. A, 30 days; B, 45 days.

17. 12.

23. Crew, 6 miles per hour; current, 2 miles per hour.

18. 24 cents.

24. 46.

Exercise 121. Page 298.

1. $x = 5$, or 2 ;

$y = 2$, or 5 .

7. $x = 6$, or 3 ;

$y = -3$, or -6 .

13. $x = -3$, or 7 ;

$y = -\frac{2}{3}$, or 4 .

2. $x = 9$, or 3 ;

$y = 3$, or 9 .

8. $x = 1$, or $\frac{2}{3}$;

$y = 3$, or $\frac{1}{3}$.

14. $x = 8$, or -12 ;

$y = 5$, or $-\frac{2}{3}$.

3. $x = 4$, or 2 ;

$y = -2$, or -4 .

9. $x = 4$, or 5 ;

$y = 0$, or 3 .

15. $x = 5$, or 4 ;

$y = 4$, or 5 .

4. $x = 11$, or -1 ;

$y = 1$, or -11 .

10. $x = -3$, or $\frac{1}{4}$;

$y = -8$, or $\frac{1}{4}$.

16. $x = \frac{7}{2}$, or 2 ;

$y = \frac{7}{2}$, or 1 .

5. $x = 8$, or 4 ;

$y = 4$, or 8 .

11. $x = 6$, or $\frac{2}{3}$;

$y = 5$, or $-\frac{1}{3}$.

17. $x = 5$;

$y = 4$.

6. $x = 5$, or -2 ;

$y = -2$, or 5 .

12. $x = 2$, or 21 ;

$y = 3$, or $\frac{2}{3}$.

18. $x = -1$, or -8 ;

$y = -6$, or -27 .

19. $x = \frac{1}{2}$;
 $y = \frac{1}{2}$.
20. $x = 2$, or 3;
 $y = 3$, or 2.
21. $x = 2$;
 $y = 3$.
22. $x = \frac{1}{2}$, or $\frac{1}{3}$;
 $y = \frac{1}{2}$, or $\frac{1}{3}$.
23. $x = 6$, or 2;
 $y = \frac{1}{2}$, or 8.
24. $x = 3$, or 2;
 $y = 2$, or 3.
25. $x = 5$, or -4;
 $y = 4$, or -5.
26. $x = 4$, or 1;
 $y = 1$, or 4.
27. $x = 5$, or 3;
 $y = 3$, or 5.
28. $x = \frac{1}{2}$, or $-\frac{1}{2}$;
 $y = \frac{1}{2}$, or $-\frac{1}{2}$.
29. $x = 5$, or 1;
 $y = 1$, or 5.
30. $x = 4$, or -2;
 $y = 2$, or -4.
31. $x = 6, 4$, or $\frac{-7 \pm \sqrt{217}}{5}$;
 $y = 4, 6$, or $\frac{-7 \mp \sqrt{217}}{5}$.
32. $x = 9, -6$, $\frac{-57 \pm 3\sqrt{-95}}{2}$;
 $y = 6, -9$, $\frac{-57 \mp 3\sqrt{-95}}{2}$.
33. $x = \pm 3$;
 $y = \pm 5$.
34. $x = \pm 8$;
 $y = \pm 7$.
35. $x = \pm 2$;
 $y = \pm 5$.
36. $x = \pm 1$, or ± 21 ;
 $y = \pm 2$, or $\mp \frac{19}{2}$.
37. $x = \pm 2$, or $\pm 4\sqrt{3}$;
 $y = \pm 5$, or $\mp \sqrt{3}$.
38. $x = \pm 2$, or $\pm \frac{8}{\sqrt{7}}$;
 $y = \pm 4$, or $\pm \frac{10}{\sqrt{7}}$.
39. $x = \pm 5$, or $\pm 6\sqrt{2}$;
 $y = \pm 2$, or $\mp 7\sqrt{2}$.
40. $x = \pm 2$, or $\pm \frac{14}{\sqrt{11}}$;
 $y = \pm 4$, or $\pm \frac{9}{\sqrt{11}}$.
41. $x = \pm 2$, or $\pm \frac{14}{7}$;
 $y = \pm 3$, or $\pm \frac{1}{7}$.
42. $x = \pm 2$, or $\pm \frac{14}{15}$;
 $y = \pm 5$, or $\mp \frac{14}{15}$.
43. $x = \frac{5 \pm \sqrt{-151}}{2}$, 3, or 2
 $y = \frac{5 \mp \sqrt{-151}}{2}$, 2, or 3.
44. $x = \frac{3 \pm \sqrt{-55}}{2}$, 2, or 1;
 $y = \frac{3 \mp \sqrt{-55}}{2}$, 1, or 2.
45. $x = 5, -4$, or $\frac{1}{2}(1 \pm \sqrt{-87})$;
 $y = 4, -5$, or $\frac{1}{2}(-1 \pm \sqrt{-87})$.
46. $x = \frac{1 \pm 3\sqrt{-3}}{2}$, 3, or -2;
 $y = \frac{1 \mp 3\sqrt{-3}}{2}$, -2, or 3.
47. $x = 3, -1$, or $1 \pm \sqrt{-6}$;
 $y = 1, -3$, or $-1 \pm \sqrt{-6}$.
48. $x = 2, 3$, or $-1 \pm \sqrt{2}$;
 $y = 3, 2$, or $-1 \mp \sqrt{2}$.
49. $x = 4$, or -2;
 $y = 2$, or -4.
50. $x = 16$, or 8;
 $y = 8$, or 16.
51. $x = 2$;
 $y = 3$.

52. $x = \pm 5$, or ± 4 .
 $y = \pm 4$, or ± 5 .
53. $x = \pm 2$, or ± 1 ;
 $y = \pm 1$, or ± 2 .
54. $x = \pm 4$;
 $y = \pm 2$.
55. $x = 4$, or 108;
 $y = 6$, or -72 .
56. $x = \pm 4$, or $-\frac{1}{2} \pm \frac{1}{2}\sqrt{65}$;
 $y = \mp 4$, or $-\frac{1}{2} \mp \frac{1}{2}\sqrt{65}$.
57. $x = 6$, or 1;
 $y = 3$, or -2 .
58. $x = \frac{1}{3}$, or $\frac{1}{4}$;
 $y = \frac{1}{4}$, or $\frac{1}{3}$.
59. $x = \pm 4$, ± 2 , or $\pm\sqrt{-\frac{4}{7}} \pm \sqrt{-\frac{1}{7}}$;
 $y = \pm 2$, ± 4 , or $\pm\sqrt{-\frac{4}{7}} \mp \sqrt{-\frac{1}{7}}$.
60. $x = 1, 2, -1$, or -2 ;
 $y = 2, 1, 0$, or -1 .
61. $x = 0$, or a ;
 $y = -a$, or 0.
62. $x = \frac{a}{2}$;
 $y = \frac{b}{2}$;
63. $x = 0, a + b$, or $\frac{a-b}{2} \pm \frac{1}{2}\sqrt{a^2 + 2ab - 3b^2}$;
 $y = 0, a + b$, or $\frac{a-b}{2} \mp \frac{1}{2}\sqrt{a^2 + 2ab - 3b^2}$.
64. $x = \pm a \pm b$;
 $y = \pm a \mp b$.
65. $x = \pm \frac{a}{b}$, or $\pm \frac{b}{a}$;
 $y = \pm \frac{b}{a}$, or $\pm \frac{a}{b}$.
66. $x = \pm \frac{a}{a+b}$, or $\pm \frac{b\sqrt{-1}}{a+b}$;
 $y = \pm \frac{b}{a+b}$, or $\mp \frac{a\sqrt{-1}}{a+b}$.
67. $x = \pm(a+b)$;
 $y = \pm(a-b)$.
68. $x = \pm \sqrt{\frac{a^2 \pm \sqrt{4b^4 + a^4}}{2}}$;
 $y = \pm \sqrt{\frac{-a^2 \pm \sqrt{4b^4 + a^4}}{2}}$.
69. $x = \pm(a+2b)$, or $\pm(a-2b)\sqrt{-1}$;
 $y = \pm(a-2b)$, or $\mp(a+2b)\sqrt{-1}$.
70. $x = a$, or b ;
 $y = b$, or a .

Exercise 122. Page 301.

- Length, 12 feet; breadth, 5 feet.
- Length, 12 feet; breadth, 9 feet.
- Length, 20 feet; breadth, 10 feet.
- Length, 24 feet; breadth, 7 feet.
- Length, 15 rods; breadth, 12 rods.
- 10; 12.
- 4; 5.
- 7; 12.
- 138.
- 5; 11.
- 9; 11.
- 73.
- 84; 48.
- $\frac{1}{2}$.
- Fore wheel, 8 feet; hind wheel, 10 feet.
- 168 square inches.
- 36 feet; 29 feet.

Exercise 123. Page 307.

1. 24 : 35 ; 3 : 5 increased ; 8 : 7 decreased.
2. 10 : 9.
4. Descending order of magnitude, 15 : 16 ; 23 : 25 ; 10 : 11 ; 3 : 4.
5. Descending order of magnitude, $a + b : a - b$; $a^2 + b^2 : a^2 - b^2$.
6. 4 : 15.
7. 2 : 3.
10. 1 : 1.
14. 4 : 1.
8. $(a^4 + a^2x^2 + x^4) : (a^4 - x^4)$.
11. 18 ; 27.
15. 1 : 5.
9. $(x^2 - 11x + 28) : x^2$.
13. 6 ; 8.
16. A to B by the shorter road, 26 miles ; by the longer road, 40 miles. B to C by the shorter road, 52 miles ; by the longer road, 60 miles.
17. $\frac{pn - mq}{q - p}$.
18. Length, 1240 rods ; breadth, 680 rods.

Exercise 124. Page 315.

1. $1\frac{1}{2}$.
14. $x = 1\frac{1}{2}$.
20. $x = \frac{a^2 - b^2}{4a - b}$.
2. ± 12 .
15. $x = 18$.
21. $x = 32$;
3. 70.
16. ± 10 ; ± 15 .
- $y = 24$.
4. $x = 1$.
17. ± 15 ; ± 25 .
22. $x = \frac{1}{3}$, or 2 ;
12. $x = 136$.
18. 100 ; 150 ; 250.
- $y = \frac{2}{3}$, or 2.
13. $x = 14$.
19. A, \$200 ; B, \$150.

Exercise 125. Page 320.

1. $x = 3$.
9. 33.5104 cubic feet.
15. $117\frac{1}{2}$ cubic feet.
2. $w = 5$.
10. $111\frac{1}{2}$ pounds.
16. $150\frac{1}{2}$ cubic feet.
5. $x = 99$.
11. 256 feet per second.
17. $14\frac{1}{2}$ cubic feet.
7. $2x = \frac{z(11z^2 + 1)}{1 - z^4}$.
12. 400 feet.
18. 6.37 feet.
13. As 1 to 2.
19. 6 miles.
8. 1256.64 square feet.
14. $84\frac{1}{2}$ cubic feet.

Exercise 126. Page 327.

1. 45.
4. $a + 21b$.
7. $4\frac{1}{3}$.
8. 6 ; a^2 .
2. 21.
5. - 1.
9. $5\frac{1}{2}$, 10, $14\frac{1}{2}$; $\frac{1}{3}$, $4\frac{2}{3}$, $8\frac{1}{3}$, $12\frac{2}{3}$.
3. - 29.
6. 8.
10. The 31st term.
11. - 7.
14. $\frac{1}{2}an(3n - 1)$.
17. 1, $2\frac{2}{3}$, $3\frac{1}{3}$, $5\frac{1}{3}$, $6\frac{2}{3}$, 8.
12. 185.
15. - 28.
18. 9.
19. - 10.
13. 35.
16. $336\frac{2}{3}$.
20. 177 feet.
21. $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{1}{3}$, $4\frac{2}{3}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{4}$, $10\frac{1}{2}$, $11\frac{1}{2}$.
22. The 10th term.
24. 105.
26. 300.
28. 4 ; 11 ; 18.
23. 6 ; 14.
25. 156.
27. 5 ; 7 ; 9.
29. - 3 ; 1 ; 5 ; 9.

Exercise 127. Page 333.

1. 1458.
2. 96.
3. $1\frac{2}{3}$.
10. 3; 192.
11. 765.
12. 547.
13. $15\frac{1}{2}$.
14. 1953.1.
25. 2, 4, 6, 9; or $8\frac{1}{2}$, $6\frac{1}{2}$, $3\frac{1}{2}$, $2\frac{1}{2}$.
28. 1, 3, 9, 27; or -81, -27, -9, -3.
4. -128.
5. $\frac{1}{4}m^4a^5$.
15. $\frac{205m}{256}$.
16. 10%.
17. 3.
18. $1\frac{1}{2}$.
20. $\pm \frac{1}{2}$, 1, ± 2 , 4.
6. $6x^2y^2\sqrt{15z}$.
7. ± 4 .
8. 20, 50; ± 28 , 56, ± 112 .
9. The 5th term.
21. $\frac{32}{243}$.
22. 12; 18; 27.
23. 1; $2\frac{1}{4}$.
24. 3; 27.

Exercise 128. Page 335.

1. 8.
2. $1\frac{1}{2}$.
3. $\frac{1}{3}$.
4. $\frac{5}{7}$.
5. $\frac{8}{9}$.
6. $\frac{1}{2}$.
7. $\frac{8}{9}$.
8. $\frac{1}{10}$.

Exercise 129. Page 337.

1. $2\frac{2}{3}$, 3, 4, 6.
2. 1 and 9; or $-7\frac{1}{2}$ and $\frac{1}{2}$.
3. 12.
8. $\frac{a+b}{2}$; $\sqrt{ab}$; $\frac{2ab}{a+b}$; the descending order of magnitude is $\frac{a+b}{2}$; $\sqrt{ab}$; $\frac{2ab}{a+b}$.
9. 104; 234.
4. $13\frac{1}{2}$, $14\frac{1}{17}$... $17\frac{1}{2}$, $18\frac{6}{13}$.
5. The sixth term.
6. 6, $6\frac{2}{3}$, $6\frac{4}{9}$, $7\frac{5}{9}$.
7. 8; 2.
10. 6; 3; 2.

Exercise 130. Page 344.

1. $-\frac{1}{2}$.
2. 1.
3. 2.
4. 2.
5. -2.
6. -1.
7. 1.
8. $2(2+\sqrt{2})$.

Exercise 131. Page 349.

1. $1 - 2x + 4x^2 - 8x^3 + \dots$
2. $\frac{1}{2} + \frac{3x}{4} + \frac{9x^2}{8} + \frac{27x^3}{16} + \dots$
3. $\frac{1}{2} - \frac{x}{4} + \frac{3x^2}{8} - \frac{9x^3}{16} + \dots$
4. $1 - 2x + x^2 + x^3 - \dots$
5. $5 - 7x + 12x^2 - 19x^3 + \dots$
6. $2 + x - 4x^2 - 11x^3 - \dots$
7. $3 + 4x + 7x^2 + 11x^3 + \dots$
8. $1 - x^2 + x^3 - \dots$
9. $1 - 7x - x^2 - 43x^3 - \dots$
10. $2 - x + \frac{x^2}{2} - \frac{x^3}{4} + \frac{x^4}{8} - \dots$
11. $\frac{2}{3} - \frac{5x}{9} + \frac{5x^2}{27} - \frac{5x^3}{81} + \frac{5x^4}{243} - \dots$

$$12. 5 - 17x + 56x^2 - 185x^3 + 611x^4 - \dots$$

$$13. -\frac{1}{2x} + \frac{1}{4} - \frac{3x}{8} - \frac{3x^2}{16} - \frac{3x^3}{32} - \dots$$

$$14. -\frac{2}{x} - 1 + x^2 + 2x^3 + \dots \quad 15. 1 - x^3 - x^4 - \dots$$

Exercise 132. Page 351.

$$1. \frac{3}{x+4} + \frac{4}{x-5}$$

$$2. \frac{5}{2x-1} - \frac{4}{3x-1}$$

$$3. -\frac{1}{3(2x-1)} + \frac{8}{3(x-5)}$$

$$4. \frac{3}{7(x-5)} + \frac{4}{7(x+2)}$$

$$5. \frac{1}{x-1} - \frac{x+2}{x^2+x+1}$$

$$6. \frac{3}{4x} - \frac{1}{8(x-2)} + \frac{3}{8(x+2)}$$

$$7. -\frac{4}{5x^2} + \frac{4}{25x} + \frac{71}{25(x+5)}$$

$$8. \frac{2}{(x-1)^2} + \frac{11}{3(x-1)} + \frac{10}{3(x+2)}$$

$$9. -\frac{4}{3(x-1)} + \frac{10x-7}{3(x^2+x+1)}$$

$$10. \frac{13}{7(6x+1)} + \frac{6}{7(x-1)}$$

$$11. \frac{4x-8}{5(x^2+1)} + \frac{1}{5(x+2)}$$

$$12. \frac{1}{2(x^2+1)} + \frac{1}{2(x-1)^2}$$

Exercise 133. Page 357.

$$1. a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

$$2. x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

$$3. 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$$

$$4. \frac{x^6}{y^6} - \frac{6x^4}{y^4} + \frac{15x^2}{y^2} - 20 + \frac{15y^2}{x^2} - \frac{6y^4}{x^4} + \frac{y^6}{x^6}$$

$$5. 256 + 768y + 864y^2 + 432y^3 + 81y^4$$

$$6. a^{21} + 7a^{18}b + 21a^{16}b^2 + 35a^{12}b^3 + 35a^9b^4 + 21a^6b^5 + 7a^3b^6 + b^7$$

$$7. m^{16} + 8m^{14}n^2 + 28m^{12}n^4 + 56m^{10}n^6 + 70m^8n^{12} + 56m^6n^{16} + 28m^4n^{18} + 8m^2n^{21} + n^{24}$$

$$8. a^7 - 7a^6b^3 + 21a^5b^6 - 35a^4b^9 + 35a^3b^{12} - 21a^2b^{15} + 7ab^{18} - b^{21}$$

$$9. a^{\frac{5}{2}} + 5a^2b^{\frac{2}{3}} + 10a^{\frac{2}{3}}b^{\frac{4}{3}} + 10ab^2 + 5a^{\frac{1}{3}}b^{\frac{8}{3}} + b^{\frac{10}{3}}$$

$$10. a^{-8} + 3a^{-2}b^{-2} + 3a^{-1}b^{-4} + b^{-6}$$

$$11. m^{-2} + 4m^{-\frac{3}{2}}n^2 + 6m^{-1}n^4 + 4m^{-\frac{1}{2}}n^6 + n^8$$

$$12. x^{-12} + 6x^{-10}z^{\frac{2}{3}} + 15x^{-8}z^{\frac{4}{3}} + 20x^{-6}z^2 + 15x^{-4}z^{\frac{8}{3}} + 6x^{-2}z^{\frac{10}{3}} + z^4$$

$$13. 32x^{10} + 80x^8y^{\frac{1}{2}} + 80x^6y + 40x^4y^{\frac{3}{2}} + 10x^2y^2 + y^{\frac{5}{2}}$$

14. $a^2 - 4a^{\frac{1}{2}}c^{\frac{3}{2}} + 6ac^{\frac{5}{2}} - 4a^{\frac{3}{2}}c^2 + c^{\frac{7}{2}}$.
15. $32a^{10} - 40a^{\frac{17}{2}} + 20a^7 - 5a^{\frac{11}{2}} + \frac{5}{2}a^4 - \frac{1}{2}a^{\frac{5}{2}}$.
16. $\frac{a^{14}}{b^7} - \frac{7a^{11}}{2b^{\frac{11}{2}}} + \frac{21a^8}{4b^4} - \frac{35a^5}{8b^{\frac{5}{2}}} + \frac{35a^2}{16b} - \frac{21b^{\frac{1}{2}}}{32a} + \frac{7b^2}{64a^4} - \frac{b^{\frac{7}{2}}}{128a^7}$.
17. $\frac{8a^8}{b^6} + \frac{6a^{\frac{5}{2}}}{b^3} + \frac{3a^2}{2} + \frac{a^{\frac{3}{2}}b^8}{8}$.
18. $16x^2y^{-4} - 32x^2y^{-\frac{3}{2}} + 24x^4y - 8x^2y^{\frac{7}{2}} + y^6$.
19. $\frac{a^{\frac{23}{2}}}{b^6} + \frac{7a^7}{b^{\frac{11}{2}}} + \frac{21a^{\frac{14}{3}}}{b^{\frac{11}{3}}} + \frac{35a^{\frac{7}{3}}}{b^{\frac{17}{3}}} + \frac{35}{b^{\frac{11}{3}}} + \frac{21b^{\frac{5}{2}}}{a^{\frac{7}{2}}} + \frac{7b^{\frac{7}{2}}}{a^{\frac{13}{2}}} + \frac{b^{\frac{7}{2}}}{a^7}$.
20. $\frac{a^{\frac{5}{2}}}{b^{\frac{10}{3}}} - \frac{5a^{\frac{3}{2}}}{2b^{\frac{13}{6}}} + \frac{5a^{\frac{1}{2}}}{2b} - \frac{5b^{\frac{1}{2}}}{4a^{\frac{1}{2}}} + \frac{5b^{\frac{4}{3}}}{16a^{\frac{2}{3}}} - \frac{b^{\frac{5}{2}}}{32a^{\frac{5}{2}}}$.
21. $128a^7b^{-14} - 448a^{\frac{13}{2}}b^{-11} + 672a^6b^{-8} - 560a^{\frac{11}{2}}b^{-5} + 280a^5b^{-2}$
 $- 84a^{\frac{9}{2}}b + 14a^4b^4 - a^7b^7$.
22. $\frac{a^6c^8}{b^6d^8} - \frac{6a^5c^{\frac{7}{2}}}{b^5d^{\frac{7}{2}}} + \frac{15a^4c^4}{b^4d^4} - \frac{20a^3c^{\frac{3}{2}}}{b^3d^{\frac{3}{2}}} + \frac{15a^2c^5}{b^2d^5} - \frac{6ac^{\frac{11}{2}}}{bd^{\frac{11}{2}}} + \frac{c^6}{d^6}$.
23. $\frac{a^{\frac{5}{2}}}{b^{\frac{5}{2}}} - \frac{5a^{\frac{3}{2}}}{b^{\frac{3}{2}}c^{\frac{1}{2}}} + \frac{10a^{\frac{1}{2}}}{b^{\frac{1}{2}}c} - \frac{10b^{\frac{1}{2}}}{a^{\frac{1}{2}}c^{\frac{3}{2}}} + \frac{5b^{\frac{3}{2}}}{a^{\frac{3}{2}}c^2} - \frac{b^{\frac{5}{2}}}{a^{\frac{5}{2}}c^{\frac{5}{2}}}$.
24. $-15120x^4y^3$.
25. $400 \cdot 33 \cdot 47 \cdot 97a^4b^{96}$.
26. $-26730x^4y^7$.
27. $-5 \cdot 19 \cdot 17 \cdot 13 \cdot 2^5a^{81}$.
28. $25 \cdot 23 \cdot 11 \cdot 2^5a^{21}b^2$.
29. $7 \cdot 11 \cdot 13 \cdot 2^9a^{10}$.
30. $3 \cdot 5 \cdot 17 \cdot 19 \cdot 2^8x^6y^{14}$.
31. $-37 \cdot 2^6 \cdot 7x^9$.
32. $-17 \cdot 13 \cdot 7 \cdot 2^8a^{23}x^{\frac{11}{2}}$.
33. $19 \cdot 37 \cdot 7 \cdot 17 \cdot 33y^{64}$.
34. $\frac{3^2 \cdot 5 \cdot 7 \cdot 19}{2^{17}}a^{17}b^6$.
35. $-1140a^{\frac{17}{2}}b^2$.
36. $-21a^{\frac{5}{2}}b$.
37. $-126a^{\frac{8}{3}}\sqrt{-1}$.
38. $\frac{19 \cdot 17 \cdot 5 \cdot 2^{\frac{7}{2}}}{9}a^{\frac{13}{2}}x^{\frac{7}{2}}$.
39. $13 \cdot 11 \cdot 10 \cdot 9x^8y^8$.
40. $5 \cdot 7 \cdot 29 \cdot 31 \cdot 3^4a^{71}b^{19}$.
41. $\frac{n(2n-1)(n-1)(2n-3)(n-2)(2n-5)}{3 \cdot 5 \cdot 6}x^{2n-13}$.

Exercise 134. Page 363.

1. $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \dots$
2. $1 - 2x + 3x^2 - 4x^3 + \dots$
3. $1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \dots$
4. $1 - \frac{2}{3}x + \frac{1}{3}x^2 - \frac{4}{81}x^3 - \dots$
5. $1 + \frac{2}{3}x + \frac{5}{9}x^2 + \frac{14}{27}x^3 + \dots$
6. $1 + \frac{5}{3}x + \frac{5}{9}x^2 - \frac{5}{81}x^3 + \dots$
7. $1 - \frac{5}{3}x + \frac{25}{9}x^2 - \frac{125}{27}x^3 + \dots$
8. $1 - 3x + 6x^2 - 10x^3 + \dots$
9. $1 - 25x + 375x^2 - 4375x^3 + \dots$
10. $1 + \frac{2}{3}x + \frac{1}{3}x^2 - \frac{8}{81}x^3 + \dots$
11. $2^{\frac{3}{4}}x^{\frac{3}{4}} + \frac{9y}{2^{\frac{3}{4}}x^{\frac{1}{4}}} - \frac{27y^2}{2^{\frac{3}{4}}x^{\frac{5}{4}}} + \frac{135y^3}{2^{\frac{3}{4}}x^{\frac{9}{4}}} - \dots$
12. $\frac{1}{2^{\frac{3}{4}}x^{\frac{3}{4}}} - \frac{9y}{2^{\frac{15}{4}}x^{\frac{7}{4}}} + \frac{189y^2}{2^{\frac{31}{4}}x^{\frac{11}{4}}} - \frac{7 \cdot 11 \cdot 27y^3}{2^{\frac{43}{4}}x^{\frac{15}{4}}} + \dots$
13. $\frac{1}{a^{\frac{1}{2}}} + \frac{x}{4a^{\frac{3}{2}}} + \frac{5x^2}{32a^{\frac{5}{2}}} + \frac{15x^3}{128a^{\frac{7}{2}}} + \dots$
14. $a^{-\frac{2}{3}} + \frac{2}{3}a^{-\frac{7}{3}}x + \frac{7}{25}a^{-\frac{12}{5}}x^2 + \frac{128}{125}a^{-\frac{17}{5}}x^3 + \dots$
15. $-\frac{27}{125}a^{-\frac{2}{3}}x^{-\frac{2}{3}}$
16. $\frac{17}{24}a^{-\frac{1}{3}}x^4$
17. $-\frac{5x^2}{4^{\frac{1}{2}}}$
18. $\frac{2}{3}a^{-\frac{2}{3}}x^5$
19. $\frac{25}{8}x^4$
20. $15x^4$
21. $-\frac{21}{1024}x^6$
22. $\frac{2n+1}{8n^2}x^2$
23. $-\frac{220}{81}x^3$
24. $\frac{207}{76 \cdot 2^{\frac{1}{2}}x^6}$
25. $\frac{7 \cdot 11 \cdot 15 \cdot 81}{2^{\frac{3}{4}}}x^{-\frac{1}{4}}y^4$
26. $\frac{27500}{81}x^3$

Exercise 135. Page 364.

1. 0.
2. $2ax + 2cy + 2bz$.
3. $5x^3 + 10x^2 - 3x - 18$.
4. $2a^{p+3} - 7a^{p+2} + 12a^{p+1} - 19a^p + 20a^{p-1} - 11a^{p-2} + 5a^{p-3}$.
5. $1 + 2x + 3x^2 + 4x^3 + 5x^4$.
6. $4h^4 - 2h^3 - 3h^2$.
7. $2a - 12b + 7c$.
8. $(10x - 1)(x + 8)$.
9. $x - 1$.
10. $\frac{a^2 - a + 5}{5a^2 + 4a + 16}$.
11. 79.
12. $(2x - 1)(3x + 4)$.
13. $(x + 1)(x + 2)(x + 3)(x - 4)$.
14. $(2x - 1)(3x + 2)(2x + 3)$.
15. 60.
16. H. C. F. = $x^2 - 2x + 1$;
L. C. M. = $(x - 1)^2(3x^3 + 6x^2 + 4x + 2)(2x^3 + 4x^2 + 6x + 3)$.
17. H. C. F. = $x + 2$;
L. C. M. = $(x + 2)(3x^2 + 4x - 1)(3x^2 + 7x + 3)$.
18. H. C. F. = $2x^2 - 3x + 1$;
L. C. M. = $(x - 1)(2x - 1)(2x^2 + 3x - 1)(3x + 1)$.

21. H. C. F. $= x^2 - 2x + 3$;
L. C. M. $= (x^2 - 2x + 3)(x - 1)(x^2 + 3x + 4)(x^3 + 2x^2 + 12x + 18)$.
22. H. C. F. $= 2x - y$;
L. C. M. $= (2x - y)(x^2 + 3xy - y^2)(x^2 - 3xy + y^2)$.
23. $\frac{5}{x(x-2)}$. 24. $\frac{4x^2}{1-x^4}$. 25. $\frac{2}{a(x+2a)}$.
26. $\frac{18}{(x-1)(x+2)(x+5)}$. 27. $\frac{x^2-8}{(x-1)(2-x)(x+3)}$.
28. $\frac{abc}{bc+ac-ab}$. 29. $\frac{4}{3x}$. 30. $x = \frac{7}{10}$;
 $y = \frac{3}{10}$.
31. $\frac{2a}{a^2+1}$. 32. $\frac{1+2a+3a^2}{1+3a+4a^2}$. 33. $x = a$;
 $y = b$;
 $z = c$.
34. 100 pounds. 35. $x = -\frac{5}{7}$. 36. $x = 3a$. 37. $x = 12$;
 $y = b$;
 $z = c$.
38. $\frac{b+1}{ab^2}$. 39. $x = 3a$. 40. $x = a$;
 $y = b$;
 $z = c$.
41. 6; 3; 2; 2; 32; 4; 243; 16. 42. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$.
43. 8; 32; 27; $\frac{8}{27}$; $\frac{2}{3}$; $\frac{1}{6}$. 44. $9\sqrt{3}$, $5\sqrt{10}$, $6\sqrt{7}$.
45. 0.5; 0.09; $\frac{1}{16}$; $\frac{1}{4}$; $\frac{1}{16}$. 46. $3\sqrt[3]{3}$, $5\sqrt[3]{2}$, $4\sqrt{6}$.
47. $\frac{14}{135}\sqrt{30}$. 48. $2 - \sqrt{3}$. 49. $2\sqrt{5} - 3$. 50. x^{58} .
51. $4\frac{2}{3}$. 52. $2 + \sqrt{3}$. 53. $\frac{1}{11}\sqrt{5} + \frac{3}{11}$. 54. x^6 .
55. $2a^{\frac{1}{2}}b^{\frac{1}{2}} + 11b$. 56. $x^{p(q-1)} - y^{q(p-1)}$. 57. $4\sqrt{5}$; $5\sqrt{5}$.
58. a^{m-1} . 59. $6\sqrt[3]{-5} + 10\sqrt[3]{5}$. 60. $3x^{-2} - 3x^{-1}y^{\frac{1}{2}} + y$.
61. x^{pq} . 62. 1. 63. $10\sqrt{2}$; $\sqrt[3]{3}$. 64. $2x + 1 - 3x^{-1}$.
65. $2^{\frac{2}{3}} \times 3^{\frac{1}{3}}$, $2^{\frac{2}{3}} \times 3^{\frac{1}{3}}$, $2^{\frac{5}{6}} \times 3^{\frac{1}{6}}$, $2^{\frac{2}{3}}$; 24.
66. $x = 12$, or -2 . 67. $x = \pm 2$, or $\pm \frac{11}{8}\sqrt{2}$;
 $y = \pm 1$, or $\pm \frac{1}{10}\sqrt{2}$.
68. $x = 4$, or $\frac{4}{3}$. 69. $x = \pm 1$, or $\pm \frac{11}{8}\sqrt{159}$;
 $y = \pm 1$, or $\pm \frac{1}{16}\sqrt{159}$.
70. $x = 6$, or $\frac{9}{4}$. 71. $x = 16$, or 8;
 $y = 9$, or 1.
72. $x = 0$, or $a + b$. 73. $x = \pm 2$, or $\pm \frac{4}{3}\sqrt{3}$;
 $y = \pm 6$, or $\pm \frac{10}{3}\sqrt{3}$.
74. $x = \pm 1$. 75. $x^2 - 2ax + a^2 - b^2 = 0$.
76. $x = \frac{3c}{a+b}$, or $-\frac{2c}{a+b}$.
77. $x^2 - (2a+b)x + a^2 + ab - 6b^2 = 0$.
78. $x^2 - 3(a+b)x + 2a^2 + 5ab + 2b^2 = 0$.
79. $x^2 - 2x - 2 = 0$. 80. $x^2 - 2x + 4 = 0$. 81. 39. 82. 234.
83. $x^2 + 2x - 2 = 0$. 84. 3 hours; 5 hours. 85. 20; 15.

86. 2, 6, 10, 14; or -2, -6, -10, -14.
88. $5\frac{1}{2}$ miles per hour.
89. $x = 2\sqrt{2}$, or $\sqrt{2}$;
 $y = \sqrt{2}$, or $2\sqrt{2}$.
90. $x = \pm \frac{1}{2}$, or ± 4 ;
 $y = \pm 5$, or ∓ 2 .
91. $x^3 - 3x + 3x^{-1} - x^{-3}$.
92. $x^{\frac{4}{3}} - 2x^2 + \frac{2}{3}x^{\frac{5}{3}} - \frac{1}{2}x^{\frac{10}{3}} + \frac{1}{18}x^4$.
93. $8a^{\frac{3}{2}} - 12a^{\frac{1}{2}} + 6a^{-\frac{1}{2}} - a^{-\frac{3}{2}}$.
94. $32x^{-10} + 80x^{-\frac{7}{2}} + 80x^{-6} + 40x^{-\frac{5}{2}} + 10 + x^{\frac{3}{2}}$.
95. $\frac{1}{18}x^2 - \frac{1}{6}x^{\frac{11}{6}} + \frac{1}{6}x^{\frac{5}{2}} - \frac{2}{7}x^{\frac{3}{2}} + \frac{1}{81}x^{\frac{4}{3}}$.
96. $\frac{1}{27}x^{-2} + \frac{1}{18}x^{-\frac{7}{3}} + \frac{1}{18}x^{-\frac{8}{3}} + \frac{1}{81}x^{-3}$.
99. 80.
100. $\frac{1}{3}$; $\frac{1}{4}$.
102. 256 feet.
101. 192 feet per second.
103. $\frac{2}{3}$ of a cubic foot.
104. 128, 32, 8, 2; or $76\frac{2}{3}$, $-19\frac{1}{3}$, $4\frac{2}{3}$, $-1\frac{1}{3}$.
106. $-\frac{1}{2}$; $-\frac{2}{3}$.
107. 0; 1.
108. $x = \pm 1$, or ± 2 .
109. $x = 1, 2, \frac{1}{2}(-1 \pm \sqrt{-3})$, or $-1 \pm \sqrt{-3}$.
110. $x = \pm 1$, or $\pm \frac{2}{3}$.
115. $x = \frac{5}{16}$, or 81.
111. $x = \pm 2$, or $\pm \frac{1}{2}$.
116. $x = (-\frac{2}{3})^3$, or 8.
112. $x = \pm \frac{1}{2}\sqrt{2}$, or $\pm \sqrt{2}$.
117. $x = \sqrt[5]{\frac{10000}{81}}$, or $\sqrt[5]{16}$.
113. $x = \frac{1}{18}$, or 1.
118. $x = 2, \sqrt[3]{\frac{3}{2}}$, or $-1 \pm \sqrt{-3}$.
114. $x = \frac{5}{729}$ or 1.
119. $x = \pm 1$, or $\pm \sqrt{-1}$.
120. $x = \pm 1, \frac{1}{2}(-1 \pm \sqrt{-3})$, or $\frac{1}{2}(1 \pm \sqrt{-3})$.
121. $x = 1$, or -729 .
127. $x = \frac{1}{2}$, or 8.
122. $x = \pm 1$, or $\pm \frac{1}{1024}$.
128. $x = 5$, or $-\frac{8}{13}$.
123. $x = \frac{2}{2401}$, or $\frac{1}{18}$.
129. $x = 3$, or $\frac{11923}{5941}$.
124. $x = -\frac{8}{729}$, or $\frac{1}{27}$.
130. $x = 7$, or $\frac{108}{61}$.
125. $x = 1$, or $\sqrt[3]{-4}$.
131. $x = 2$, or $-\frac{1}{174}$.
126. $x = a^6$, or $\frac{a^6}{729}$.
132. $a^{\frac{2}{3}} - 9a^4x^{\frac{2}{3}} + 36a^{\frac{7}{3}}x^{\frac{4}{3}} - 84a^3x^2 + 126a^{\frac{5}{3}}x^{\frac{8}{3}} - 126a^2x^{\frac{10}{3}} + 84a^{\frac{8}{3}}x^4$
 $- 36ax^{\frac{14}{3}} + 9a^{\frac{1}{3}}x^{\frac{16}{3}} - x^6$;
 $243a^{\frac{5}{3}} - 810a^{\frac{4}{3}}b^2 + 1080ab^4 - 720a^{\frac{2}{3}}b^6 + 240a^{\frac{1}{3}}b^8 - 32b^{10}$;
 $a^{\frac{1}{3}} - \frac{1}{2}a^{\frac{1}{3}}b^{\frac{1}{2}} + \frac{2}{3}a^{\frac{1}{3}}b - \frac{3}{8}a^{\frac{1}{3}}b^{\frac{3}{2}} + \frac{3}{16}a^{\frac{1}{3}}b^2 - \frac{3}{32}a^{\frac{1}{3}}b^{\frac{5}{2}} + \frac{7}{64}a^{\frac{1}{3}}b^3 - \frac{1}{128}b^{\frac{7}{2}}$;
 $64 - 64x^2 + \frac{8}{3}x^4 - \frac{16}{27}x^6 + \frac{2}{9}x^8 - \frac{4}{81}x^{10} + \frac{1}{729}x^{12}$;
 $2187 - \frac{5103}{4}x^{\frac{1}{3}} + \frac{5103}{4}x - \frac{2835}{8}x^{\frac{2}{3}} + \frac{245}{16}x^2 - \frac{189}{8}x^{\frac{4}{3}} + \frac{3}{4}x^3 - \frac{1}{128}x^{\frac{5}{2}}$;
 $x^{16} - 8x^{12} + 28x^8 - 56x^4 + 70 - 56x^{-4} + 28x^{-8} - 8x^{-12} + x^{-16}$.

133. $x^{10} - \frac{5}{2}x^8y^2 + \frac{5}{2}x^6y^4 - \frac{5}{2}x^4y^6 + \frac{5}{2}x^2y^8 - \frac{1}{1024}y^{10}$;
 $16x^8 + 32x^6\sqrt{3}x + 72x^5 + 24x^3\sqrt{3}x + 9x^2$;
 $a^{15} + 10a^{11} + 10a^9 + 5a^7 + 10a^5 + 5a^3$
 $+ \sqrt{a^2+1}(5a^{12} + 10a^8 + 10a^6 + a^4 + 2a^2 + 1)$;
 $1 + 6x + 9x^2 - 7x^3 - 21x^4 + 14x^5 + 3x^7 - 3x^8 - x^9$.
134. $1 + x + 2x^2 + \frac{1}{3}x^3 + \dots$;
 $1 + \frac{2}{3}x^2 + \frac{8}{9}x^4 + \frac{256}{81}x^6 + \dots$;
 $1 - x^{\frac{1}{2}} - \frac{1}{4}x - \frac{1}{8}x^{\frac{3}{2}} - \dots$;
 $a^{-5} - 10a^{-6}x^{-\frac{1}{2}} + 60a^{-7}x^{-\frac{3}{2}} + 280a^{-8}x^{-1} + \dots$.
135. $15 \cdot 16 \cdot 22 \cdot 89 \cdot 87x^4y^{86}$.
136. $\frac{4}{1-2x} - \frac{1}{1-x}$; $-\frac{1}{2(1-x)} + \frac{7}{2(1-3x)}$; $\frac{1}{3(1-x)} + \frac{x+2}{3(1+x+x^2)}$
137. $3 + 7x + 15x^2 + 31x^3 + 63x^4 + \dots$.

Exercise 136. Page 381.

- | | | |
|------------------|-------------------|-----------------|
| 1. 3.79029. | 11. 1.26784. | 21. 1.28544. |
| 2. 9.79029 - 10. | 12. 7.62328 - 10. | 22. 296.8. |
| 3. 3.45743. | 13. 5.14006 - 10. | 23. 77,770,000. |
| 4. 1.93328. | 14. 7.09562 - 10. | 24. 816.98. |
| 5. 2.47246. | 15. 8.80232 - 10. | 25. 18.528. |
| 6. 3.84535. | 16. 7.06084 - 10. | 26. 6170. |
| 7. 9.93922 - 10. | 17. 4.81248 - 10. | 27. 154,000. |
| 8. 9.77144 - 10. | 18. 8.15463 - 10. | 28. 0.617. |
| 9. 4.86477. | 19. 2.61925. | 29. 0.0042003. |
| 10. 1.82815. | 20. 3.10958. | 30. 0.00014277. |

Exercise 137. Page 384.

- | | | |
|----------------|---------------|-----------------|
| 1. 665.66. | 11. 42.093. | 21. 0.14434. |
| 2. 0.32168. | 12. 0.66992. | 22. 1.1649. |
| 3. 0.78523. | 13. - 3.7834. | 23. 1.1634. |
| 4. 7.7113. | 14. 3.0621. | 24. 127.26. |
| 5. - 49.292. | 15. 4486.2. | 25. 761,900. |
| 6. - 3.8164. | 16. 0.6354. | 26. 34,878,000. |
| 7. - 0.38238. | 17. 176.52. | 27. 75.917. |
| 8. 43.1. | 18. 794.98. | 28. 0.31264. |
| 9. 14,646,000. | 19. 101.95. | 29. 0.84948. |
| 10. 0.69838. | 20. 301.81. | 30. 0.00048689. |

- | | | |
|------------------|--------------|--------------|
| 31. 0.000073033. | 37. 0.98488. | 42. 2.16096. |
| 32. 1.7353. | 38. 6.7927. | 43. 1.89570. |
| 33. 0.39895. | 39. 0.47047. | 44. 5.46998. |
| 34. 0.38735. | 40. 1.0089. | 45. 1.19897. |
| 35. 0.8089. | 41. 1.43068. | 46. 6.57845. |
| 36. 37.601. | | |

Exercise 138. Page 388.

- | | | |
|--------------------------------|--------------------------------|----------------|
| 1. \$13,596. | 4. \$1483.50. | 7. \$707.50. |
| 2. \$29,558. | 5. \$852.80. | 8. 4%. |
| 3. \$19,210. | 6. \$11,706. | 9. 5%. |
| 11. 10 years 9 months 14 days. | 12. 12 years 2 months 24 days. | 10. 4% |
| 13. \$7037.80. | 15. \$9828.80. | 17. \$17,550. |
| 14. \$9043.80. | 16. \$34,946. | 18. \$8078.80. |

WENTWORTH'S MATHEMATICS

By GEORGE A. WENTWORTH.

Wentworth's Elementary Arithmetic.....	\$0.30
Wentworth's Practical Arithmetic.....	.65
Wentworth's Primary Arithmetic.....	.30
Wentworth's Grammar School Arithmetic.....	.65
Wentworth's Advanced Arithmetic.....	1.00
Wentworth's Mental Arithmetic.....	.30
Wentworth and Hill's Exercises in Arithmetic (complete).....	.80
Wentworth and Reed's First Steps in Number:	
Teachers' Edition (complete).....	.90
Pupils' Edition.....	.30
Wentworth's First Steps in Algebra.....	.60
Wentworth's School Algebra.....	1.12
Wentworth's New School Algebra.....	1.12
Wentworth's Higher Algebra.....	1.40
Wentworth's Elements of Algebra.....	1.12
Wentworth's Complete Algebra.....	1.40
Wentworth's Shorter Course in Algebra.....	1.00
Wentworth's College Algebra.....	1.50
Wentworth and Hill's Exercises in Algebra (complete).....	.70
Wentworth's Plane Geometry.....	.75
Wentworth's Plane Geometry (Revised).....	.75
Wentworth's Plane and Solid Geometry.....	1.25
Wentworth's Plane and Solid Geometry (Revised).....	1.25
Wentworth's Solid Geometry (Revised).....	.75
Wentworth's Plane and Solid Geometry (Revised) and Plane Trig.....	1.40
Wentworth's Analytic Geometry.....	1.25
Wentworth's Logarithms and Metric Measures.....	.20
Wentworth's Geometrical Exercises.....	.10
Wentworth's Syllabus of Geometry.....	.25
Wentworth and Hill's Examination Manual in Geometry.....	.50
Wentworth and Hill's Exercises in Geometry.....	.70
Wentworth's New Plane Trigonometry.....	.40
Wentworth's New Plane Trigonometry and Tables.....	.90
Wentworth's New Plane and Spherical Trigonometry.....	.85
Wentworth's New Plane and Spherical Trigonometry, with Tables.....	1.20
Wentworth's New Plane Trigonometry and Surveying, with Tables.....	1.20
Wentworth and Hill's Logarithmic and Trigonometric Tables.....	50 & 1.00
Wentworth's New Plane and Spherical Trigonometry, Surveying, with Tables.....	1.35
Wentworth's New Plane and Spherical Trigonometry, Surveying, and Navigation.....	1.20

GINN & COMPANY, Publishers,

Boston. New York. Chicago. Atlanta. Dallas.

WENTWORTH'S GEOMETRIES

REVISED EDITIONS

By GEORGE A. WENTWORTH,
Author of a Series of Text-Books in Mathematics.

Wentworth's Plane Geometry. REVISED.

Half morocco. 256 pages. 75 cents.

Wentworth's Plane and Solid Geometry. REVISED.

Half morocco. 473 pages. \$1.25.

Wentworth's Solid Geometry. REVISED.

Half morocco. 210 pages. 75 cents.

IN recognition of the remarkable success of Wentworth's Geometry for the past twenty years, the author and publishers have done their best to make this revised edition a model text-book. The typography and geometrical figures are all that can be desired. The proofs are rigorous and at the same time simple. The exercises are sufficient in number and arranged with the greatest care. Methods of attacking and proving theorems and of solving problems are explained, and illustrated by examples worked out in full. Many exercises not found in the old edition have been added.

Special attention is invited to the development of the principles of limits and their application to the propositions of geometry that involve limits. Special attention is also called to the finely executed woodcuts that are used to illustrate the definitions of solid geometry, and to assist the student to cultivate a correct geometrical imagination.

GINN & COMPANY, Publishers,

Boston. New York. Chicago. Atlanta. Dallas.

WENTWORTH'S ALGEBRAS

FOR USE IN HIGH SCHOOLS, ACADEMIES, AND COLLEGES.

By GEORGE A. WENTWORTH,
Author of Wentworth's Series of Mathematics.

THE extraordinary success of Professor Wentworth's text-books on algebra must have a cause: the cause assigned by those in the best position to judge is that the books are *just right*. The author has aimed for the center of the shield and has struck it.

Remember the leading characteristics of these remarkable books, — scholarship and text-book availability. They combine mathematical brilliancy with all the elements of school-room popularity.

Wentworth's First Steps in Algebra. For Elementary Classes.
Half leather. 184 pages. For introduction, 60 cents.

Wentworth's New School Algebra. For High and College Preparatory Schools. Half morocco. 407 pages. For introduction, \$1.12.

Wentworth's Higher Algebra. For Normal Schools and Colleges.
Half morocco. 528 pages. For introduction, \$1.40.

Wentworth's College Algebra. For Colleges and Technical Schools. Half morocco. 500 pages. For introduction, \$1.50.

Wentworth's Trigonometries. Plane and Spherical, Surveying, and Navigation with Tables, bound variously.

For the titles of the various editions of Wentworth's Trigonometries, prices for introduction, etc., see our High School and College Catalogue and our Wholesale Price List.

GINN & COMPANY, Publishers,

Boston. New York. Chicago. Atlanta. Dallas.

ADVANCED ARITHMETIC

By **WILLIAM W. SPEER,**

District Superintendent of Schools, Chicago.

Part III. For Grammar Grades.

12mo. Cloth. Illustrated. 261 pages. Price, 50 cents.

SPEER'S ADVANCED ARITHMETIC, — the third book in the series is in harmony with the theory and practice of the first two volumes. The means of advance relied upon are still the sensory, intellectual, and expressive powers of the learner. Formal statements and demonstrations, which deprive the pupil of personal expression and conceal his mental condition, are superseded by opportunity for expression which is self-revealing. The presentation is such that the pupil advances in the indirect establishment of relations of magnitude by living in practical contact with mathematical realities. With attention centered upon the relative magnitudes, he is led to think in symbols rather than about them.

Speer's Arithmetics advance the study from the science of number to that of definite relations of quantity; they break down the artificial barriers built between groups of problems that are alike in process and differ only in trade usage. They recognize sense-training as the basis of thought, and definite relations of magnitudes as the only basis of mathematical inferences.

They make **simple ratios the key** to the solution of all problems, and thus unify all parts of the work and save much time; they unfold the pupil's mind, not only by leading him to perceive, but also to form judgments and to reason.

GINN & COMPANY, Publishers,

Boston. New York. Chicago. Atlanta. Dallas.

TEXT-BOOKS ON GEOMETRY

By GEORGE A. HILL,

*Formerly Assistant Professor of Physics in Harvard University,
and Associate Author of Wentworth and Hill's
"Text-Book of Physics."*

Hill's Geometry for Beginners. 12mo. Cloth. 320 pages. For introduction, \$1.00.

Hill's Lessons in Geometry. For the use of beginners. 12mo. Cloth. 190 pages. For introduction, 70 cents.

THE **Geometry for Beginners** presents the subject in the natural method as distinguished from the formal method of Euclid, Legendre, and the common text-books. The central purpose is intellectual training, or, teaching by practice how to think correctly and continuously.

The **Lessons in Geometry** presents a shorter and easier course than that given in the "Geometry for Beginners."

It is designed to prepare the mind for the logical study of the subject in the ordinary text-books, yet made complete in itself for the sake of those who are able to give only a brief period of time to the study of geometry.

The method of this book is not a formal but a natural method, suited to the mental condition of the average boy twelve to fourteen years old. It recognizes the absence of the geometric imagination, and the inability to put together even the simplest links of reasoning so that they shall form a logical chain.

Geometric models are studied at the start, and the imagination is exercised by asking questions which the learner can answer by reflecting on what he has seen. The first steps in consecutive reasoning are not taken till considerable time has been spent in the inductive study of form and magnitude.

An important part of the method consists in requiring easy constructions to be made, which involve drawing to scale and the use of ruler and compasses.

BY THE SAME AUTHOR:

Hill's Drawing Case. Prepared expressly to accompany Hill's "Lessons in Geometry," and containing, in a neat wooden box, a *seven-inch rule* with a scale of millimeters; *pencil compasses*, with pencil and rubber; a *triangle*; and a *protractor*. For introduction, 30 cents.

A specimen copy of the "Lessons in Geometry" with the "Drawing Case" will be sent, postpaid, to any teacher on receipt of \$1.00.

GINN & COMPANY, Publishers,

Boston. New York. Chicago. Atlanta. Dallas.

TEXT-BOOKS ON MATHEMATICS

FOR HIGHER SCHOOLS AND COLLEGES.

Anderegg and Roe's Trigonometry.....	\$0.75
Bailey and Woods' Analytic Geometry.....	2.00
Baker's Elements of Solid Geometry.....	.80
Beman and Smith's Higher Arithmetic.....	.80
Beman and Smith's Famous Problems of Elementary Geometry..	.50
Beman and Smith's New Plane and Solid Geometry.....	1.25
Byerly's Differential Calculus.....	2.00
Byerly's Integral Calculus.....	2.00
Byerly's Fourier's Series.....	3.00
Byerly's Problems in Differential Calculus.....	.75
Carhart's Field-Book.....	2.50
Carhart's Plane Surveying.....	1.80
Faunce's Descriptive Geometry.....	1.25
Hall's Mensuration.....	.50
Hanus' Determinants.....	1.80
Hardy's Elements of Quaternions.....	2.00
Hardy's Analytic Geometry.....	1.50
Hardy's Differential and Integral Calculus.....	1.50
Hill's Geometry for Beginners.....	1.00
Hill's Lessons in Geometry.....	.70
Hyde's Directional Calculus.....	2.00
Osborne's Differential Equations.....	.50
Peirce's (B. O.) Newtonian Potential Functions.....	1.50
Peirce's (J. M.) Elements of Logarithms.....	.50
Peirce's (J. M.) Mathematical Tables.....	.40
Runkle's Plane Analytic Geometry.....	2.00
Smith's Coördinate Geometry.....	2.00
Taylor's Elements of the Calculus (Revised and enlarged).....	2.00
Tibbets' College Requirements in Algebra.....	.50
Wentworth's Advanced Arithmetic.....	1.00
Wentworth's School Algebra.....	1.12
Wentworth's New School Algebra.....	1.12
Wentworth's Higher Algebra.....	1.40
Wentworth's College Algebra.....	1.50
Wentworth's Plane Geometry.....	.75
Wentworth's Plane Geometry (Revised).....	.75
Wentworth's Plane and Solid Geometry.....	1.25
Wentworth's Plane and Solid Geometry (Revised).....	1.25
Wentworth's Solid Geometry (Revised).....	.75
Wentworth's Plane and Solid Geometry (Revised) and Plane Trig.	1.40
Wentworth's Analytic Geometry.....	1.25
Wentworth and Hill's Five-place Log. and Trig. Tables (7 tables)	.50
Wentworth and Hill's Five-place Log. and Trig. Tables (complete edition).....	1.00
Wheeler's Plane and Spherical Trigonometry and Tables.....	1.00

GINN & COMPANY, Publishers,

Boston. New York. Chicago. Atlanta. Dallas.







